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IX.—SYSTEMS OF ELECTRICAL MEASUREMENT.

By F. E. Smith, *C.B.E., F.R.S.*

Introduction.

In the first place I wish to thank you for the honour you have done me in re-electing me as your President. For many years I have had the interests of the Society very much at heart, and I shall endeavour to serve it faithfully during the coming year.

Presidential addresses are in general of two types ; one a review of the history of the Society, and the other a review of some special branch of science. I have decided on the latter type and propose to review a branch of physics, the last chapter of which many regard as having been written. I refer to the C.G.S. System of Electrical Measurements. In my review I shall bear in mind the objects of the Society, more especially the "showing by new means of old facts" ; I shall endeavour to put before you certain ideals of Kelvin and Maxwell and other great physicists, who had the subject of units and standards very much at heart ; I shall try by means of experiments to show you that the C.G.S. System of Electrical Measurements is simple and not beyond the understanding of a good student of elementary physics ; my review will show that for the past 60 years there has been a continual struggle between a scientific ideal and a practical demand ; that struggle is still going on, and I propose to indicate a solution of the difficulty.

It is possible that the future will see a serious competitor of the C.G.S. system, for I believe it likely that there will arise a complete system of measurements based on the electron as the unit of electric quantity. Measurements of electric quantity in terms of the electron have been made for many years, but for practical purposes as visualised at present an electron system would not, I think, have any considerable advantages over the C.G.S. system.

I do not propose to review in detail the history of the C.G.S. system of electrical units. I must, however, emphasize certain features in the history, and in particular I shall concentrate on the following points.

(1) When the units based on the centimetre, gramme and second were recommended it was fully realised that the definitions and statements accompanying them were too complex to become popular definitions. In the first Report of the British Association Committee on Electrical Units and Standards, the following statement appears :—

"It was apparent that, although a foot of copper or a metre of mercury might not be very scientific standards, they produced a perfectly definite idea in the minds of even ignorant men and might possibly, with certain precautions, be both permanent and reproducible, whereas Weber's unit has no material existence, but is rather an abstraction than an entity."

This statement is, I believe, almost equally true to-day and I shall endeavour this evening to overcome this objection to some extent by experiments showing how easy it is to measure resistance and current in terms of the centimetre, gramme and second.

(2) The second point I wish to draw attention to is the fact that we never have had standards identical with the units. In 1863 the Committee stated :—

"The practical adoption of the absolute system was felt to depend on the accuracy with which the absolute resistance of some one standard conductor could be measured."

And in another part of the Report the Committee state :—

"The only hesitation felt by the Committee was caused by doubts as to the degree of accuracy with which this admirable system could be or had been reduced to practice."

It is most important to note that unless a unit can be realised within limits of error not likely to affect industry there is no alternative to setting up a separate standard. A separate standard was set up in 1861, and the practice has continued until the present time. The B. A. Committee were greatly impressed by the advantages of the C.G.S. system, but they felt the units could not be realised sufficiently closely for practical purposes. In this they were correct. Their first attempt to realise the unit of resistance was in error by more than 1 per cent. The actual requirements of telegraph engineers at that time I do not know, but the comparative conducting powers of metals were being measured within 1 part in 10,000.

I shall emphasize the fact that the unit and standard have always differed.

(3) The third point is that not only have the unit and standard differed, but the standard has been changed several times, and another change will shortly be necessary. I shall endeavour to show that this change should be final and should result in the units and standards becoming practically identical.

EARLY ELECTRICAL SYSTEMS.

Before, however, proceeding with these main points, it is of interest to examine the nature of electrical measurements before the C.G.S. system was introduced. In the fifties of the nineteenth century, electrical engineering was practically confined to telegraphy ; the transmission of signals by electromagnetic apparatus was suggested in 1821 by Ampere and shortly after this Gauss and Weber constructed a telegraph line over a distance of nearly two miles. In 1831, Henry produced a successful system for transmitting signals, but it was left to Morse in 1844 to establish the first long distance line, from Baltimore to Washington. Experiments on submarine telegraphy began in 1837, and the first Atlantic cable expedition began in 1857. In the eighteen-fifties it may be truly said that electrical engineering consisted of telegraphy. It is true there were dynamos, but it was not until 1866 that electromagnets were used in dynamo machines. It is not strange, therefore, that the names of some of the pioneers of the electric telegraph—Ampère, Gauss, Weber, and Henry—should be associated with electric units.

EARLY ELECTRICAL STANDARDS.

Sixty years ago resistances were expressed in terms of a length of telegraph wire, and voltages were given in terms of the number of batteries used. In England, the most commonly used standard of resistance was a mile of No. 16 gauge copper wire ; in France it was a kilometre of iron wire 4 mm. in diameter ; in Russia a copper wire 1 metre long and 1 mm. in diameter was used, and in Germany, Siemens' Unit, which was 1 metre length of mercury 1 sq. mm. in cross-section, was employed.

In addition to these Wheatstone proposed and used 1 ft. of copper wire weighing 100 grains. Resistance coils based on Wheatstone's standard were used by the Electric and International Telegraph Company. Two such standard coils are on the table ; these were made before 1861.

The following table shows some of the resistance standards previous to 1861 :—

Name.	*Nature.*
Jacobi ...	25 ft. of copper wire, weighing 345 grammes.
Siemens ...	1 metre of mercury, 1 sq. mm. cross section.
Digney ...	1,000 metres of iron wire, 4 mm. in diameter.
Matthiessen ...	1 mile of pure annealed copper wire, $\frac{1}{16}$in. diameter.
Varley ...	1 mile of a special copper wire, $\frac{1}{16}$in. diameter.
Wheatstone ...	1 ft. of copper wire, weighing 100 grains.
German Mile ...	8,238 yds. of iron wire, $\frac{1}{6}$ in. in diameter.

With regard to standards of voltage there were three, the Daniell Cell, which was invented in 1836, the Grove, invented in 1839, and the Bunsen, which was described in 1841. Voltages were usually expressed as a number of Daniell cells. There were also batteries of a simple type of which Davy in one experiment used 2,000.

There were no ammeters, the only current-measuring devices being galvanometers, and results were either recorded as deflections of these, or the wire and batteries used were stated. For submarine telegraphy measurements, a standard of capacity was needed ; in some cases a sea mile of Persian Gulf cable was the reference standard, and in other cases a sea mile of Atlantic cable. Such was the position in 1861.

BRITISH ASSOCIATION COMMITTEE.

It will be realized that in 1861, when the British Association appointed a Committee to improve the "Construction of Practical Standards for Electrical Measurements," the position can only be described as chaotic. Lord Kelvin served on this Committee for 34 years, Maxwell for 7 years, Lord Rayleigh for 27 years, and there were many other distinguished men, some of whom fortunately are still with us. The great result of the labours of this Committee was the adoption by the civilized world of a system of electrical measurements based on the centimetre, gramme, and second as the fundamental units ; it is known as the C.G.S. system, or absolute system. In their Report of 1863, the Committee state :—

"The absolute system is, however, not only the best practical system, but it is the only rational system. Everyone will readily perceive the absurdity of attempting to teach geometry with a unit of capacity so defined that the contents of a cube should be 6¼ times the arithmetical cube of one side, or with a unit of surface of such dimensions that the surface of a rectangle would be equal to 0·000023 times the product of its sides ; but geometry so taught would not be one whit more absurd than the science of electricity would become unless the absolute system of units were adopted."

THE C.G.S. SYSTEM.

In the C.G.S. system of electrical measurements the fundamental units are the centimetre, gramme and second, but as is now generally recognised, there is nothing in their nature which entitles them to be termed "absolute." In passing, I wish to point out that some alternative standard to the present international metre —a rod of platinum-iridium—is needed, and it is gratifying to note that at the last meeting of the International Committee on Weights and Measures (Metric) a resolution was adopted accepting in principle the eventual adoption of the wavelength of light as the ultimate reference standard of length.

What was it that led to the first proposal of a system of measurements based on three fundamental units ? It appears that Gauss was engaged in measuring magnetic force and realising that all forces may be measured by the motions they produce, it was apparent that all forces could be measured in the same units, viz. a unit of length, of mass, and of time. Gauss therefore proposed a system of magnetic measurements based on these units. His colleague Weber in 1851 extended the idea, and proposed a general system of electromagnetic measurements based on the same three fundamental units. Lord Kelvin was greatly interested in the proposals which had been made, and extended them, and it was mainly due to him that the British Association Committee was appointed.

In electrical measurements we are concerned with current, resistance, voltage, quantity, force, work and so on. The real nature of many of these quantities is unknown, but Gauss found "force" to serve as a link between mechanics and magnetism, and Kelvin pointed out that "work" serves as a link connecting all branches of measurement in physical science. The dimensions of force are MLT^{-2}, and of work ML^2T^{-2}.

In electricity it is known that if two currents flow in two wires a force is set up which tends to move the wires. The common example is that of two coils through which the same current flows ; a force of attraction or repulsion is set up. Hence the dimensions of current are $M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$.*

Again, we know that if a current flows through a resistance work is done, and the work done is proportional to the square of the current, the resistance and the time. Hence Work$=C^2Rt$, and the dimensional equation must be

$$ML^2T^{-2}=MLT^{-2}\times\frac{L}{T}\times T.$$

That is, the dimensions of resistance are the same as those of velocity. To measure a resistance it is therefore only necessary to measure a particular length and a particular time. Kelvin illustrated resistance being measured as a velocity in a manner somewhat as follows :—

Resistance as a Velocity.

Suppose a wire of any material to be bent into an arc of one radian, the radius being r cm. If a current i flows through this arc, the intensity of the magnetic field at the centre of the arc is (by definition of the unit of current)

$$\frac{i}{r}$$

Let the arc of radius r be in the plane of the magnetic meridian with the upper extremity vertically over the lower one, and let two parallel wires or a continuous chain r centimetres apart join the ends as shown in Fig. 1. (The plane of the paper is supposed to be at right angles to the magnetic meridian.)

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* This assumes that the permeability μ of the medium has no dimensions. This is for convenience, but really the dimensions of μ are unknown.

Let the extreme ends be joined by a metallic rod L, and PP' be two rollers which when pulled in the direction X cause L to travel with a velocity V. Then the rate of cutting the lines of the magnetic field is HVr, and the current in the circuit is $\frac{HVr}{R}$, where R is the resistance of the circuit. Let the velocity be altered until the intensity of the field produced by the current in the arc is equal to H—i.e., a magnetic needle suspended at the centre of the arc is deflected 45°. Then, since

$$H=\frac{i}{r}$$

and

$$i=\frac{HVr}{R}$$

$$V=R$$

In other words, the velocity of the rod in centimetres per second is equal to the resistance of the circuit.

This experiment is not a convenient one to make, but an adaptation is easy, Such an adaptation is the well-known B.A. rotating coil devised by Kelvin. The principle of this is very simple, and if the self-inductance is neglected the calculation is easy for students.

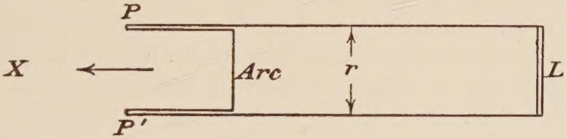

FIG. 1.

Consider a circle of wire spinning about a vertical axis in the earth's field with a magnet suspended at its centre. If A is the area of the circle, the average electromotive force is $\frac{4AH}{T}$, where T is the time of one revolution. The maximum E.M.F. is $\frac{\pi}{2}$ times this, i.e., $\frac{2\pi AH}{T}$, and the instantaneous E.M.F. in any position is

$$\frac{2\pi AH \sin \theta}{T}$$

$(90°-\theta)$ being the angle between the plane of the coil and the meridian.

The instantaneous current is

$$\frac{2\pi\, AH \sin \theta}{RT}$$

where R is the resistance of the ring. This current produces a magnetic field at the centre having a component at right angles to the meridian equal to

$$\frac{2\pi}{r} \cdot \frac{2\pi AH \sin^2 \theta}{RT}$$

The average value of this for a complete cycle is

$$\frac{2\pi^2 AH}{r\,RT}$$

Hence the needle will be deflected through an angle φ such that

$$\tan\varphi=\frac{2\pi^2 A}{rRT}$$

and since

$$A=\pi r^2$$

$$R=\frac{2\pi^3 r}{T\tan\varphi}$$

It was not possible for the B.A. Committee to use a single turn, for a single turn is of low resistance, and accurate methods of comparing a very low resistance with others of medium value were unknown then. Later Lord Kelvin invented the double bridge for such comparisons. However, to-day, in good electrical laboratories, it is comparatively easy to compare a resistance of nominal value 0·001 ohm with another of 1 ohm, and in standardising laboratories such a comparison can be made with an error not greater than 1 part in 1,000,000.

ABSOLUTE MEASUREMENTS OF *R*.

I stated that one of my objects was to show by means of experiment that it is not difficult to measure to a first approximation an electric quantity in terms of C.G.S. units. I propose to demonstrate two methods for the measurement of resistance, one the method of a rotating coil or ring, and the other a mutual inductance method.

In spirit let us imagine we are back with Kelvin and the others in 1862. The practical units are fixed ; it remains to see if we can realise our ideals.

I proceed to measure the resistance of this ring by the B.A. method of a rotating coil.

The mean radius r is 12·2 cm.
$2\pi^3 r$ is 756 cm.
T and φ by observation we find to be 0·1 second and 2·4 degrees respectively.
Hence the resistance R is $1{\cdot}8\times10^5$ cm. per second, or 0·00018 ohm.

The principle of the experiment is simple ; it should be understood by any electrical engineer, for it is the simplest form of rotating armature.

In subsequent absolute measurements determinations were made by methods involving a mutual inductance. Kirchoff, Rowland, Glazebrook, Roiti and Himstedt and others used mutual inductances in their experiments. I propose to make a second measurement of resistance using a mutual inductance method that is very simple.

From the definition of unit current we know that the intensity I of the magnetic field at the centre of a circular coil is

$$I=Ci$$

C being equal to $\frac{2\pi n_1}{r_1}$ (the coil constant).

r_1 is the radius of the coil, n_1 the number of turns, and i is the current.

If a much smaller coil of effective area A ($A=\pi r_2^2 n_2$, where n_2 is the number of turns) is placed co-planar and co-axial with the large coil, and if a fluxmeter or ballistic galvanometer is placed in series with the small coil, the quantity Q_1 of electricity passing through the circuit containing the small coil when the current through the larger one is made or broken is approximately

$$Q_1=\frac{CAi}{R} \quad \dots\dots\dots\dots\dots \quad (1)$$

(the assumption is made that the field is uniform over the cross-section of the smaller, coil, an assumption only approximately true if the second coil is much the smaller one). In the example which follows the error is about 1 per cent. If in a second experiment the circuit of the larger coil includes the fluxmeter and is made for a short time t, the quantity Q_2 of electricity passing through the fluxmeter is

$$Q_2=it \quad \dots\dots\dots\dots\dots \quad (2)$$

Combining (1) and (2) we have

$$R=\frac{Q_2}{Q_1} \cdot \frac{CA}{t}$$

By adjusting t we may make $Q_1=Q_2$, and so eliminate any error due to calibration of the fluxmeter.

R is in absolute measure if the radii of the coils are in centimetres and t is in seconds.

In the apparatus before you

$r_1=36{\cdot}0$ cm.
$n_1=200$
$r_2=6{\cdot}0$ cm.
$n_2=200$

$C=34{\cdot}9$, $A=22{,}600$ and $CA=790{,}000$ cm.

With this apparatus, if Q_2 is made approximately equal to Q_1, the time t is of the order of 2×10^{-5} second. It is exceedingly difficult to measure such a short interval of time with accuracy, but it is not really necessary if a known fractional part of the current is used in the second experiment.

Fig. 2 shows the circuit I am using. In the circuit of the larger coil two resistances in parallel are included: the ratio of these resistances is 1/100,000 (their actual values need not be known), and in the second experiment the fluxmeter F is included in the high resistance branch of the circuit for a time t. Through this branch the current is $i/100{,}000$. Equation (2) becomes therefore

$$Q_2=\frac{it}{100{,}000}$$

and

$$R=\frac{Q_2}{Q_1} \cdot \frac{CA}{t} \cdot 10^5$$

In practice a fine fuse wire W_1 is included in the branch circuit, and a second fuse wire W_2 effectively shunts the fluxmeter. A simple pendulum with a heavy bob carrying a strong projecting wire is set swinging, the period being t. As it swings it is caused to break the fuse wire W_2, thus causing the current $i\times10^{-5}$ to flow through the fluxmeter. During the next swing it breaks the fuse wire W_1, thus causing the current through the fluxmeter to cease. The arrangement is such that the interval is exactly t, the period of the pendulum.

The period of the pendulum before you is 2·3 seconds. Q_2/Q_1 is measured by the fluxmeter to be 1·28; hence

$$R = 1{\cdot}28 \times 790{,}000 \times \frac{10^5}{2{\cdot}3} \text{ cm. sec.}^{-1}$$

$$= 4{\cdot}4 \times 10^{10} \text{ cm. sec.}^{-1}$$

$$= 44 \text{ ohms } (10^9 \text{ C.G.S. units}).$$

I hope these two experiments suffice to show that it is not difficult to make absolute measurements of resistance with simple apparatus, and with formulæ involving little more than a knowledge of the definition of the units.

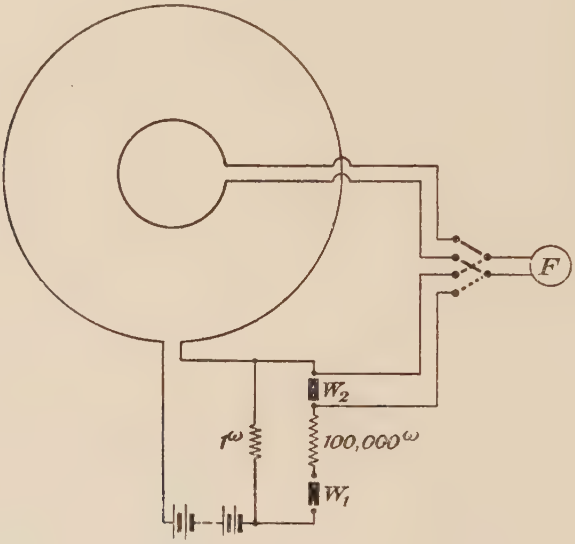

Fig. 2.

Unit and Standard.

I now propose to deal with my second principal point—namely, that we never have had standards identical with the units.

On the first page of their first Report the Committee drew an important distinction between a unit and a standard. It is necessary to realise that a unit need not have, and many units have not, material existence. However, in choosing their units the B.A. Committee specified that they should be perfectly definite, and should not be liable to require correction or alteration from time to time. The C.G.S. units comply with this specification, and no change has been made in them. The *standard* of resistance was, however, regarded as a material thing, and this also was not to be subjected to correction from time to time. It has, however, been subject to several changes, and it is desirable to consider the effect of this.

In the case of the International Standard and Unit of length the metre may be regarded as having been fixed first and the scientific unit—the centimetre—afterwards. But it is obvious that if the standard of length changes due to changes in the substance of which it is made the unit changes in the same proportion. The same is true for the international standard and unit of mass. These standards and units must be regarded as material things. Distinctly different is the unit of time, which is based on the rotation of the earth. The scientific unit of time—i.e., the interval in which the earth rotates through 1/86,400 part of the angle which it turns through in a mean solar day—cannot be represented by any material thing. The standard second corresponds to this unit, but for practical purposes standard clocks are used. If the second obtained through the medium of a standard clock changes, the unit is unaffected. In practice difficulties do not arise, because by continued astronomical observations the standard is made to correspond to the unit within limits quite negligible for all practical purposes.

As a general statement, therefore, it appears that when the standard and unit are material things, standard and unit change together; but when the unit is not a material thing any instrument constructed to realise it may change, and if the instrument is regarded as a standard, difficulties may arise unless continual checks are made. It is because the fundamental electrical units are analogous to the second, inasmuch as they are not material things, and also because for many years it was not possible to make the standards correspond to the units within limits negligible for all practical purposes, that it has been necessary to have numerous conferences to consider electrical units and standards.

The fundamental C.G.S. electrical units can never be material things, because time enters into the dimensions of all of them. Those of resistance are LT^{-1}, of current $L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$, and of electromotive force $L^{3/2}M^{\frac{1}{2}}T^{-2}$. The B.A. Committee chose as practical units

For Resistance	10^9	absolute units
,, Voltage	10^8	,, ,,
,, Current	10^{-1}	,, ,,

UNIT AND STANDARD OF RESISTANCE.

After devising apparatus to realise the unit of resistance, and after deciding on the best form and material for the standard to represent the unit,* there were two courses open to the Committee—

(1) To fix the limits within which the standard represented the unit and to recommend more experiments with a view to reducing the difference between the standard and the centimetre, second basis. As the limits of error were reduced the value of the standard would be expressed in an increasing number of significant figures.

(2) To regard the first determination as sufficient for all practical purposes for a long period, the standard being the one first constructed whether or not it agreed with the centimetre, second basis.

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* By the unit is meant 10^9 abs. units for resistance and 10^{-1} abs. units for current.

We may *imagine* a sequence of determinations as shown in the following table:—

Year in which absolute measurements were made.	Probable error of the absolute measurements.	Value in C.G.S. units of a material standard.	Errors permissible in industrial measurements.	
			(*a*)	(*b*)
1861	±1 per cent.	0·99	±0·1 per cent.	±1·0 per cent.
71	±0·5	$0{\cdot}98_5$	±0·1	±1·0
81	±0·1	0·988	±0·1	±1·0
91	±0·05	$0{\cdot}988_5$	±0·01	±0·1
1901	±0·01	0·9887	±0·01	±0·1
11	±0·005	$0{\cdot}9887_5$	±0·001	±0·01
21	±0·001	0·98873	±0·001	±0·01

Consider the 1861 figures. The difficulty of realising the C.G.S. unit at that time was such that subsequent measurements might alter the value by 1 per cent., whereas industry would be affected by a change of value of one-tenth of this amount.

Suppose that at the present time the nominal value of the standard of resistance was changed by 1 per cent. Then in testing electric lamps for candle-power the watts supplied would be changed by 1 per cent. and the candle-power per new watt would be changed by about 10 per cent. Such a change of nominal value of the standard would clearly be a grave matter. On the other hand, if the nominal value of the standard of resistance were changed by 0·05 per cent. the candle-power per new watt would be different by about 0·5 per cent. and might be a negligible amount to industry.

In general if the industrial position is that shown in column 4 the standard to be used for industrial purposes will possibly be appreciably different from the C.G.S. unit, for it will not be easy to change the nominal value of the standard. On the other hand, if the industrial position is that shown in the last column of the table, the nominal value of the standard may be changed so as to accord with the results of measurements of smaller probable error than the preceding ones.

The real position in 1861 is not clear to me, but the Committee decided, no doubt wisely, that the standard was not to be subject to correction from time to time, even though it differed appreciably from the unit. The Committee in their first report state :—

"Few defects could be more prejudicial than this continual shifting of the standard. This objection would not be avoided even by a determination made with greater accuracy than is expected at present, and was considered fatal to the unqualified adoption of the absolute unit as the standard of resistance."

It was agreed, however, that the material standard should be such as should ensure the most absolute permanency and should represent 10^9 cm./sec. as nearly as possible, and with the advance of science the difference between this standard and the 10^9 cm./sec. basis should be ascertained with increased accuracy in order that corrections could be applied by those who required them, but the material standard was under no circumstances to be altered in substance or definition.

In 1864 wire standards were made and issued, but it was proposed that the values of these should not be called "absolute measure," but that the name 1862 unit or B.A. unit, or "Ohmad," should be adopted.

From the commencement, therefore, it was realised that the unit and the standard might differ by an amount significant for practical purposes, and this position is true to-day. In 1861 the unit and the standard differed by 1 per cent., to-day the unit and standard differ by 0·05 per cent. Because of a difference between the C.G.S. unit and the standard many absolute measurements have been carried out since 1861. I do not propose to describe these, but the following list is of interest.

Date.	Observer.	Ohm in cm. of mercury.
	Method of Rotating Coil.	
1863	B. A. Committee	...
1881	Rayleigh and Schuster	...
1882	Rayleigh	...
1882	H. Weber	...
	Mutual Inductance Method.	
1878	Rowland	...
1883	Glazebrook	...
1884	Rowland	106·34
1884	Roiti	105·896
1886	Himstedt	106·08
	Weber's Method of Transient Currents.	
1885	Wiedemann	106·162
1885	Mascart, de Nerville and Benoit	106·37
	Weber's Method of Damping.	
1884	Wild	106·027
1888	F. Kohlrausch	106·34
1889	Dorn	106·243
	Method of Lorenz.	
1873	Lorenz	...
1883	Rayleigh and Sidgwick	...
1884	Rowland, Kimball and Duncan	106·29
1889	Duncan, Wilkes and Hutchinson	...
1891	V. Jones	106·307
1913	Smith	106·245
	Method of Alternating Current.	
1912	Campbell	106·273
1924	Campbell	106·243
	Gruneisen's Method.	
1920	Gruneisen and Giebe	106·246

UNIT AND STANDARD OF CURRENT.

I now turn to the second fundamental unit, the Ampère. In this case no material standard is possible, but nevertheless much the same procedure has been adopted, and a standard current has been specified which either deposits silver at a certain rate when passed through a silver voltameter or causes a specified force to be exerted between two coils forming part of a balance.

ABSOLUTE MEASUREMENT OF CURRENT.

The absolute measurement of electric currents by their mutual action on one another was explained very simply by Maxwell in 1863. Maxwell's two coils were a long way apart, and their planes at right angles to each other. Weber suggested two coils, a large and a smaller one suspended centrally, the axes of the two coils being inclined. The first current weigher or current balance was used by Joule, and consisted of two flat co-axial coils.

The coils described by Maxwell are the equivalent of magnetic shells, and Maxwell's description led me to build a small coil—somewhat like a modern wireless inductance—with a comparatively small moment of inertia, but capable of carrying a large current. The coil I am about to experiment with is the same I used in the mutual inductance method for the measurement of resistance; it weighs 80·5 grammes, has an effective radius of 6·0 cm., consists of 200 turns of enamelled wire, and a current of 2 ampères can be passed through it without undue heating. This coil is equivalent to a magnetic shell of moment Ai, where i is the current and A is the total effective area ($n\pi r^2=22{,}600$). The moment of inertia of the system is 1,450 C.G.S. units.

This small coil is set swinging at the centre of a large coil carrying the same current i; the time of swing of the small coil about a vertical axis is

$$t=2\pi\sqrt{\frac{I}{AiF}}$$

where I is the moment of inertia of the small coil, and F is the intensity of the field at the centre. F is equal to Ci, where C is the coil constant ($2\pi n/r$). Hence

$$t=2\pi\sqrt{\frac{I}{ACi^2}}$$

that is,

$$i=\frac{2\pi}{t}\sqrt{\frac{I}{AC}}$$

I propose to use the same large coil as I used in the measurement of resistance. The plane of the coil is in the magnetic meridian. For this coil $C=34{\cdot}9$.

Hence—

$$i=\frac{2\pi}{t}\sqrt{\frac{1{,}450}{22{,}600\times 34{\cdot}9}}$$

$$=\frac{0{\cdot}269}{t}\text{C.G.S. unit.}$$

I proceed to make an experiment. The time of swing only has to be determined. t is found to be 1·9 seconds. Hence

$$i=0{\cdot}142 \text{ C.G.S. unit.}$$

$$=1{\cdot}42 \text{ amperes } (10^{-1} \text{ C.G.S.}).$$

A correction for the intensity H of the earth's horizontal magnetic field may be made by measuring the change in the position of rest of the suspended coil when the current through both coils is reversed. If 2θ is the change in the angular position, $\tan\theta=H/F$ and the resultant field acting on the suspended coil is $F/\cos\theta$. The corrected formula for the current is

$$i=\frac{2\pi}{t}\sqrt{\frac{I\cos\theta}{AC}}$$

In the experiment just made the correction is about 4 parts in 10,000.

CURRENT WEIGHER.

By far the most direct and instructive way of measuring current is to measure the force between two coils carrying the current and various forms of current weighers have been devised to do this.

The form of current weigher I propose to experiment with is simple. It consists of a large coil of many turns with a smaller coil supported at its centre, the planes of the two coils being at right angles, and the smaller being free to swing like the beam of a balance, the axis of motion being in the plane of the large coil. It is similar in principle to Pellat's current weigher.

If a current i flows through both coils the intensity I of the field at the centre of the large coil is

$$Ci,$$

where C is the coil constant as before.

The small coil is equivalent to a magnetic shell of moment

$$Ai,$$

A being the effective area as before, and the couple acting on the coil is

$$ACi^2.$$

This is balanced by a mass m at distance l from the centre. Hence

$$mgl = ACi^2,$$

or

$$i^2 = \frac{mgl}{AC}$$

In the balance I have constructed the knife "edges" consist of wires of the cobalt steel known as "stellite." The "planes" of the balance are also of stellite, and the current passes into and out of the coil by means of these wires and planes. The large coil is the one I have already used with constant C ($C = 2\pi n/r$), equal to 34·9; this is the intensity of the field at the centre of the coil when unit current flows through the coil.

The small coil is 4·7 cm. in effective radius and has 200 turns. It is equivalent to a magnetic shell having a moment A ($A = \pi r^2 n$) of 13,900 C.G.S. units.

The product AC is $48{\cdot}4 \times 10^4$ C.G.S. units.

The length l of an arm of the balance is 5 cm., and $g = 981$ cm./sec.^{2}.

Hence

$$i^2 = \frac{m \times 981 \times 5}{484{,}000}$$

or

$$i = \frac{\sqrt{1{\cdot}01m}}{10} \text{ C.G.S. units.}$$

In the experiment I choose $m = 1$ gramme, and adjust the current through the coils until the beam is balanced. The current is then $0{\cdot}100_5$ C.G.S. unit or $1{\cdot}00_5$ ampere.

STANDARDS OF CURRENT.

Current has been and is frequently measured in C.G.S. units through the tangent and sine galvanometers, the horizontal intensity of the earth's magnetism being measured by the method of Gauss. There are no material standards of current, but a current weigher has been erected by the Board of Trade as a standard, and the silver voltameter has been specified as a standard instrument. A combination of a standard cell with a standard resistance also serves as a standard of current. Any variations in a standard current weigher such as that of the Board of Trade arises because of changes in the dimensions of parts of the balance. Any variations in a standard such as a silver voltameter occur because of our lack of knowledge of the actions which take place, or they may be due to changes of procedure. Such variations can be made very small.

The relation between the C.G.S. unit of current and certain standards has been measured by many experimenters ; the majority of these results are given in the following table :—

RESULTS OF ABSOLUTE MEASUREMENTS OF CURRENT.

Date.	Observer.	Results given by Author.		
		Electro-chemical equivalent of silver in mgm.	E.M.F. of Clark cell.	E.M.F. of Weston Normal cell at 20°·0 C.
	1.	*Method of Tangent Galvanometer.*		
1873	F. Kohlrausch	1·1363	......	...
1886	F. and W. Kohlrausch...	1·11833	......	...
1904	Van Dijk and Kunst ...	1·1180	......	...
1910	Haga and Boerema ...	1·11802	......	1·01826*
	2.	*Method of Sine Galvanometer.*		
1872	Latimer Clark ...	......	1·4562† B.A. volts at 15°·5C.	...
1886	T. Gray	1·118	......	...
	3.	*Electrodynamometer Methods.*		
1872	Latimer Clark ...	......	1·4573† B.A. volts at 15°·5C.	...
1898	Patterson and Guthe ...	1·1192	......	...
1899	Carhart and Guthe ...	......	1·4333* at 15°·0 C.... ...	...
1906	Guthe	1·11773	1·43296* at 15°·0 C. ...	1·01853*
1913	Shaw	......	......	1·01827*
	4.	*Current Weigher Methods.*		
1882	Mascart...	1·1156	......	...
1884	Rayleigh	1·11794	1·435 at 15°·0 C.	...
1890	Pellat and Potier ...	1·1192	......	...
1908	Ayrton, Mather & Smith	1·11827	......	1·01818*
1908	Janet, Laporte and Jouast	1·11821	......	1·01836*
1908	Guillet	......	......	1·01812*
1908	Pellat	......	......	1·01831*
1910	Smith	1·11815	......	1·01818*
1911	Rosa, Dorsey, Miller ...	1·11804	......	1·01822*
1914	Shaw	......	......	1·01831*

Various forms of voltameters were used, and the numbers given for the electrochemical equivalent do not enable a good comparison to be made.

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* The E.M.F. is expressed in terms of the ampere (10^{-1} C.G.S.) and the international ohm.

† The E.M.F. is expressed in terms of the ampere (10^{-1} C.G.S.) and the B.A. unit.

PROBABLE CHANGES.

My final point is that not only have the electrical standards been varied in the past because they have differed from the units, but another change must be made shortly.

At present the standard of resistance differs from the unit by about 5 parts in ten thousand; the standard of electromotive force differs from the unit by about the same amount, while the standard of current is in close agreement with the unit of current (10^{-1} C.G.S.).

The differences are too great. A large number of physical measurements depend on the fundamental electrical standards, and there are many reasons why our standards should be in closer agreement with the C.G.S. units.

In 1881 the B.A. Committee reported :—

"It might be well to reconsider the question whether the 'ohm' should be defined by reference to a particular coil of wire preserved as a concrete standard, or whether the term 'ohm' should be understood to mean a resistance of 10^9 C.G.S. units."

In 1881 this was not possible as the errors in realising the unit were too great. To-day the difficulties have been swept aside by the work of the great national laboratories of Great Britain, the United States, France, Germany and Japan, and I venture to suggest that the next change should be for the units and standards to be identical, the ohm to be 10^9 C.G.S. units and the ampere to be 10^{-1} C.G.S. unit, the limits of error to be stated. At present resistance and current can be measured in C.G.S. units within a few parts in 100,000, perhaps 1 part in 100,000. The international units cannot be realised any closer than this, and actually the work of realising the international units is the more expensive.

At present the National Physical Laboratory has apparatus for measuring resistance and current in C.G.S. units, and work is in progress to make the current balance more precise and easier to manipulate. The Bureau of Standards has a fine current balance, and for this I understand new coils are to be built. At the Bureau there are at present two methods being experimented with for the absolute measurement of resistance. Both of these methods make use of stationary coils, the mutual inductance or self-inductance of which can be computed from measured dimensions. Since the coils are stationary, the dimensions can readily be determined with great accuracy. Moreover, the effect of external magnetic fields will be negligible. In one method the drop in potential over a resistance is compared with the electromotive force induced in the secondary of the mutual inductance. This comparison is made directly without the use of a condenser. In the second method, the self-inductance is compared with a condenser, whose value has been determined in terms of the international ohm.

In France excellent work has been done on the absolute measurement of current, and at the Reichsanstalt apparatus exists for the absolute measurement of resistance.

In the next few years I believe the change will come, and the international standards will be displaced, for they must be regarded, in the words of Lord Rayleigh, as a "fifth wheel to the coach."

X.—AN INVESTIGATION INTO CORRECTIONS INVOLVED IN THE MEASUREMENT OF SMALL DIFFERENCES IN REFRACTIVE INDEX OF DISPERSIVE MEDIA BY MEANS OF THE RAYLEIGH INTERFEROMETER, WITH SPECIAL REFERENCE TO THE APPLICATION OF THE RESULTS TO MEASUREMENTS IN DIFFUSION.

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Abstract.

An account is given of the use of a Rayleigh interferometer in a series of subsidiary experiments conducted in connexion with a recent determination of the diffusivity of solutions of salts in water.

The theory of the instrument, as affected by the different dispersions of the various media traversed by the light-rays, is studied, and the observations are corrected so as to apply to the green mercury light $\lambda = 5461\text{Å}$, which was employed in the experiments on diffusion.

§ 1. Introduction.

In the course of a recent investigation on Diffusion in Liquids by an optical method (Proc. Phys. Soc., v. 36, p. 313, 1924), it was necessary to know the value of the rate of variation $(d\mu/dn)$ of the refractive index, for green mercury light $\lambda = 5461\,\text{Å}$, of a salt solution with respect to its concentration, for the different salts investigated in that research, and for various strengths of those solutions.

A considerable amount of work has been done on the connexion between the refractive index of an aqueous salt solution and its concentration, and it was at first anticipated that sufficient data would be found published to supply the information required.

It is, however, evident that to obtain $(d\mu/dn)$ to an accuracy of, say, 1 per cent., a considerably higher accuracy is necessary in the values of μ, and on comparing the published results with one another, it was found that this degree of accuracy was often not realised in the researches carried out 20 or 30 years ago, and still quoted as standard results.

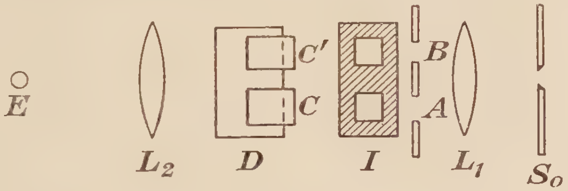
See Wagner (Tables for Dipping Refractometer, Sondershausen, 1907).
Bender (Wied. Ann., v. 39, p. 89, 1890), (Ann. d. Phys., v. 8, p. 109, 1892).
Schütt (Zeit. f. ph. Chem., v. 5, p. 349, 1890).
Dinkhauser (Wien. Ber., v. 114, p. 1001, 1905).
Borgesius (Wied. Ann., v. 54, p. 221, 1895).
Hallwachs (Wied. Ann., v. 47, p. 380, 1892).
Dijken (Ziet. f. ph. Chem., v. 24, p. 81, 1897).

A comparison of the results of these researches will show that, for the refractive index of solutions of the same salt, under similar conditions of concentration, temperature, &c., variations of a unit in the fourth decimal are not uncommon, and an error of this magnitude causes an error of about 2 per cent. in the differential $(d\mu/dn)$.

Moreover, apparently no experimenter has employed the mercury arc as his luminous source in these investigations, and in most cases it is impossible to correct the published results so as to apply to $\lambda = 5461$ Å, owing to insufficient observations having been made to permit the necessary interpolations.

It was discovered, too, that in some cases very few results were available for solutions of some of the salts whose diffusivity had been experimented upon (e.g., potassium nitrate).

In order to obtain the required results in these cases, and, even where sufficient data are otherwise available, to avoid the inaccuracies liable to be associated with interpolation for both temperature and wavelength, it was decided that it would be advisable to make a special determination of the quantity under consideration $(d\mu/dn)$, under conditions similar to those which hold in the diffusion experiments; and of the various methods of experiment at our disposal, it was resolved to employ an interferometer of the Rayleigh type (Proc. Roy. Soc., v. 59, p. 202, 1896).

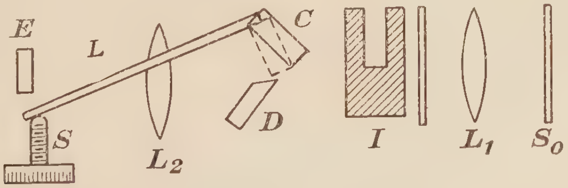


Plan

Fig. 1.

§ 2. Apparatus.

The general arrangement of the apparatus is shown in Figs. 1 and 2, the source of light being a 100-volt, 60-watt, gas-filled electric lamp. Fiducial fringes, produced by the pencils of light which pass through the apertures *A* and *B* (about 3 mm. wide and 1 cm. apart) and the glass base of the interferometer cell *I* are seen in the lower part of the field of view of the eyepiece *E*, which, in accordance with Lord Rayleigh's suggestion, is made of a short piece of glass rod about 3 mm. diameter set with its axis vertical, in order to give the magnification necessary to observe the fringes due to such widely separated sources as *A*, *B*.



Side Elevation

Fig. 2.

Two solutions of known small difference in concentration are placed respectively in the two parts of the double cell *I*, and are traversed by the two beams of light.

The resulting interference fringes are displaced with respect to the fiducial comparison fringes, and, in general, entirely disappear from view with the source of light employed, as the difference in the optical paths through the two solutions is too great.

The compensating plate C' (Figs. 1 and 2) was fixed at about 45° to the horizontal, while C was mounted on a horizontal axis, so as to be capable of rotation by means of a lever L and the screw S, of pitch 1/50th of an inch, the head of which was divided into 200 parts, so that its least count was 0·0001 in.

By rotating this screw, the effective thickness of the compensating plate C can be varied, and so the sum of the optical paths through the solution and the compensator may be made equal for the two beams passing through A and B respectively.

When this is the case, the interference fringes are seen to appear in the upper part of the field of view also, and the " achromatic " fringe can be adjusted so as to correspond exactly in position with the similar " achromatic " fringe seen in the lower part of the field.

The reading r of the screw S is then taken. The zero reading r_0 of the screw can similarly be found when the interferometer cell contains similar solutions in both parts, and $(r-r_0)$, the difference between the screw readings, enables us to calculate the difference $d\mu$ in the refractive indices of the two solutions under investigation, whose concentration-difference is dn.

§ 3. THEORY OF THE INTERFEROMETER.

The effect of dispersion in the solutions and the compensating plates is frequently neglected in using the interferometer to measure the refractive index of solutions and similar media, and the theory connected with this aspect of the problem seems to have been little studied.

L. H. Adams (Jour. Wash. Acad. Sci., v. 5, p. 265, 1915) has given some account of the theory involved, but he appears to make certain assumptions which can hardly be accepted completely. For instance, he says : " If (as in actual practice) the solution has shifted these fringes N fringes to the left, and the compensator an equal number to the right . . ." (loc. cit., p. 275).

This in general does not occur in actual practice, for the shorter optical path through the diluter solution is increased by altering the effective amount of glass in that path by a rotation of the compensator plate. The relative dispersions of the two solutions and of glass and air cause a " wandering " of the achromatic fringe, i.e., the achromatic fringe moves relatively to the interference bands as the plate is rotated, so that there is no guarantee that the plate has shifted the fringes by an exactly equal and opposite amount to the displacement caused by the solutions.

The treatment of the problem given by R. W. Wood (" Physical Optics ") and by C. R. Mann (" Manual of Advanced Optics ") refers to the Michelson interferometer, and is not easily applicable to the present instrument.

A more complete investigation of the theory would appear to be as follows :— Consider rays from two similar sources A and B (Fig. 3) at a distance a apart, falling on a screen OQ. Let rays AO and BO pass respectively through solutions of refractive indices μ_1 and μ_2 for light of a wavelength λ, and also through glass plates of refractive index μ inclined at an angle i to the rays, where i is in the neighbourhood of 45°.

The optical path difference introduced by the plate,

$$P=\mu AB-AC \quad (\textit{see } \text{Fig. } 4)$$
$$=\mu e/\cos r-(e/\cos r)\cos(i-r),$$

which simplifies to

$$P=e[\mu\cos r-\cos i] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where e is the thickness of the compensating plate, and i and r are the angles of incidence and refraction respectively.

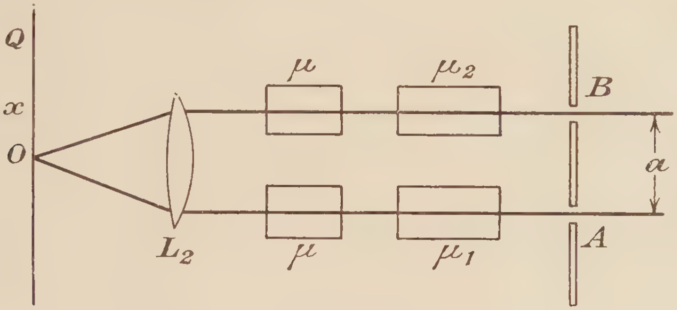


FIG. 3.

If $H=2\mu^2\ \ 1$,

$i=45°+\theta$, where θ is a small angle. Then

$$P=(e/\sqrt{2})\,[\sqrt{H-\sin 2\theta}-\cos\theta+\sin\theta] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Similarly, the optical path difference introduced by the solution is $E(\mu_1-1)$ and $E(\mu_2-1)$ respectively, where E is the length of the cell of the interferometer traversed by the rays.

If the same solution is in both parts of the interferometer cell and the compensating plates of equal thickness e are exactly parallel to one another, the phase of waves of wavelength λ from A (Fig. 3) lags behind that of waves from B by an amount $(2\pi/\lambda)(ax/f)$, where x is measured from the centre O of the screen towards Q, and f is the focal length of the lens L_2 (Figs. 1, 2 and 3).

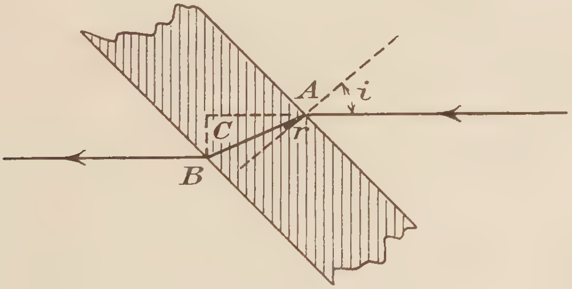


FIG. 4.

This phase difference is zero at the point where $x=O$ for all wavelengths, so that the central fringe at O is achromatic under the conditions mentioned; but it is increased both by a difference in the concentration of the solutions in the cell and by a rotation of the compensating plates by amounts $(2\pi/\lambda)E(\mu_1-\mu_2)$ and $(2\pi/\lambda)(P_1-P_2)$, where the suffixes refer to the paths of waves from A and B respectively.

Hence the total phase difference between the waves from A and B at any point x on the screen OQ (Fig. 3) is given by

$$\varphi=\frac{2\pi}{\lambda}\left\{\frac{ax}{f}+E(\mu_1-\mu_2)+(P_1-P_2)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now at the *central fringe* of the displaced system—that is, the fringe of zero path difference, $\varphi=o$.

If x_0 denotes the position of this displaced central fringe,

$$x_0=(f/a)[E(\mu_2-\mu_1)-(P_1-P_2)], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which gives a general expression for the position of the central fringe for any two solutions in the cell of refractive indices μ_1 and μ_2, and for any inclinations of the compensating plates suggested by the quantities P_1 and P_2.

It will be observed that before the plates are turned to produce compensation—i.e., while $P_1=P_2$—the displaced position of the central fringe (*see* Fig. 5), produced by the solutions alone, is given by—

DISPLACED CENTRAL FRINGE.

$$x'_0=(f/a)E(\mu_2-\mu_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

These equations (4) and (4*a*) indicate a displacement of the central fringe which varies with the colour of the light employed, since the refractive indices vary with the wavelength, so that, using white light, the central fringe will be displaced by different amounts for the different colours and will in general be no longer achromatic. Indeed there will be no strictly achromatic fringe at all in the displaced fringe system, but an approximately achromatic line, which will not in general coincide with the central fringe, will be obtained at the position where the change in phase-difference with change in wavelength is zero.

That is, *for achromatism*, $d\varphi/d\lambda=o$. Therefore, from equation (3),

$$ax/f+E(\mu_1-\mu_2)+(P_1-P_2)=E\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)+\lambda d(P_1-P_2)/d\lambda.$$

If x_1 denotes the position of this "achromatic line" in the general case,

$$x_1=(f/a)[E(\mu_2-\mu_1)-(P_1-P_2)+E\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)+\lambda d(P_1-P_2)/d\lambda]. \quad . \quad . \quad (5)$$

The condition for achromatism will not in general be satisfied at a position which coincides with one of the maxima of the interference fringes observed along OQ (Fig. 3), for which the condition holds that φ is an integral number of times 2π, for the brightest component of the light used.

In experimental work, that particular fringe will be selected as most nearly achromatic, which is nearest to the theoretical achromatic line given by $d\varphi/d\lambda=o$.

The values of (P_1-P_2) and of $d(P_1-P_2)/d\lambda$ may be found by putting $\sin\theta=0$ and $\cos\theta=1$, in equation (2). Thus we find

$$(P_1-P_2)=(e/\sqrt{2H})(\sqrt{H}-1)(\theta_1-\theta_2)\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$d(P_1-P_2)/d\lambda=[2\mu(P_1-P_2)/H(\sqrt{H}-1)](d\mu/d\lambda). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

DISPLACED ACHROMATIC LINE.

Before compensation, $P_1=P_2$, and the displaced position of the "Achromatic Line" (*see* Fig. 5) is given by

$$x'_1=(f/a)[E(\mu_2-\mu_1)+E\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)] \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

A consideration of equations (4*a*) and (5*a*) will show that for the optical media employed in this investigation, the displacement x'_1 of the achromatic line produced by the solutions alone (i.e., with $P_1=P_2$) is greater than x'_0, that of the central fringe under the same conditions.

Now, suppose that the displacement of the fringes produced by the difference in concentration of the solutions is "compensated" by a rotation of one or both of the plates, so that the "achromatic line" is restored to its undisplaced position (*see* Fig. 5), and suppose that (p_1-p_2) is the special value of (P_1-P_2) required for this compensation. It is to be observed that in practice we cannot with certainty perform this operation, for the achromatic *line* may differ in position from the achromatic *fringe* by an amount not greater than half a fringe-width, and it is only the achromatic fringe which can be identified in experimental work.

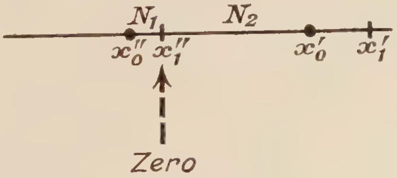


FIG. 5.

If, however, the compensation suggested could be exactly performed, then equation (5) becomes

ACHROMATIC LINE RESTORED.

$$x''_1=o=(f/a)[E(\mu_2-\mu_1)-(p_1-p_2)+E\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)$$
$$+\lambda d(p_1-p_2)/d\lambda]$$

i.e., $E(\mu_2-\mu_1)-(p_1-p_2)=-[E\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)+\lambda d(p_1-p_2)/d\lambda]$ (8)

And from equation (4),

SIMULTANEOUS POSITION OF CENTRAL FRINGE.

$$x''_0=(f/a)[E(\mu_2-\mu_1)-(p_1-p_2)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4b)$$

$$=-(f/a)[E\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)+\lambda d(p_1-p_2)/d\lambda] \quad . \quad (9)$$

which gives the position of the central fringe at the time when the "achromatic line" has been restored to its original undisplaced position, by a rotation of the compensator plates.

An inspection of equation (9) will demonstrate that for ordinary kinds of glass, when the displacement of the achromatic line has been exactly compensated by a rotation of one of the plates, that of the central fringe is *over* compensated—i.e., the movement of the screw *S* (Fig. 2) has been in general greater than is necessary to restore the central fringe to its undisplaced position, and as x''_0 is found to be numerically less than $(x'_1-x'_0)$ (*see* Fig. 5), it follows that the dispersion in the glass introduced to effect the compensation partly counter-balances the dispersion of the solutions.

Again, in order to simplify these equations for practical application, suppose that it were practicable to turn the compensating plates so that the *central fringe* were undisplaced, and let (p_3-p_4) be the special value of (P_1-P_2) required for this compensation. Then equation (4) would become

$$0=(f/a)[E(\mu_2-\mu_1)-(p_3-p_4)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4c)$$

$$\text{i.e., } E(\mu_2-\mu_1)=p_3-p_4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This adjustment of the plates, which would enable us readily to find $(\mu_2-\mu_1)$, is unfortunately not practicable, since we have no means of identifying the central fringe. The " achromatic " fringe is the only one which can be identified.

Then, from equations (9) and (4*a*), we have

$$\begin{aligned} -x''_0/x'_0 &= [E(\lambda d\mu_1/d\lambda-d\mu_2/d\lambda)+\lambda d(p_1-p_2)/d\lambda]/E(\mu_2-\mu_1) \\ &= [1/(\mu_2-\mu_1)][\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)+(\lambda/E)\,.\,d(p_1-p_2)/d\lambda], \quad . \quad . \quad . \quad (11) \\ &= [1/(\mu_2-\mu_1)][\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)]+2\mu/[H(\sqrt{H}-1)](\lambda d\mu/d\lambda) \\ &\qquad [(p_1-p_2)/(p_3-p_4)] \quad . \quad . \quad . \quad (11a) \end{aligned}$$

from equations (7) and (10).

This gives the ratio of the over-compensated displacement $(-x''_0)$ of the central fringe when the " achromatic line " has been restored to its undisplaced position, to the total displacement (x'_0) of the central fringe produced by the solutions under investigation before compensation. Now, if N_1 denotes the number of fringe-widths for a wavelength λ through which the " achromatic line " has been shifted relatively to the central fringe, in the process of compensation just referred to, and N_2 denotes the total number of fringe-widths in the displacement of the central fringe by the solutions before any compensation has been effected, then

$$N_2\lambda=ax'_0/f=E(\mu_2-\mu_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and $N_1/N_2=-x''_0/x'_0$ (*see* Fig. 5). Also,

$$\frac{p_1-p_2}{p_3-p_4}=\frac{E(\mu_2-\mu_1)+E\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)+\lambda d(p_1-p_2)/d\lambda}{E(\mu_2-\mu_1)} \text{ from equations (8) and (10)}$$

$$=1+[1/(\mu_2-\mu_1)][\lambda(d\mu_1/d\lambda-d\mu_2/d\lambda)+(\lambda/E)\,.\,d(p_1-p_2)/d\lambda]$$

$$=1+N_1/N_2 \text{ from equation (11).}$$

Substituting these values in equation (11*a*), we have

$$\frac{N_2}{N_1+N_2}=\left[1-\frac{2\mu}{H(\sqrt{H}-1)}\,.\,\lambda\frac{d\mu}{d\lambda}\right]\Big/\left[1+\frac{1}{\mu_2-\mu_1}\,.\,\lambda\,.\left(\frac{d\mu_1}{d\lambda}-\frac{d\mu_2}{d\lambda}\right)\right] \quad . \quad . \quad (13)$$

Now let us assume

$$\begin{aligned} \mu &= A+B\lambda^{-2} & &\text{i.e., } \lambda d\mu/d\lambda=-2B/\lambda^2. \\ \mu_1 &= A_1+B_1\lambda^{-2} & &\text{i.e., } \lambda d\mu_1/d\lambda=-2B_1/\lambda^2. \\ \mu_2 &= A_2+B_2\lambda^{-2} & &\text{i.e., } \lambda d\mu_2/d\lambda=-2B_2/\lambda^2. \end{aligned}$$

Then, from equation (13)

$$\frac{N_2}{N_1+N_2}=\left[1+\frac{2}{\lambda^2}\cdot\frac{2\mu(\mu-1)}{H(\sqrt{H}-1)}\cdot\frac{B}{\mu-1}\right]\Big/\left[1+\frac{2}{\lambda^2}\cdot\frac{B_2-B_1}{\mu_2-\mu_1}\right] \quad . \quad . \quad . \quad (14)$$

Now for the spectrum lines F and C, expressing λ in microns,

$$\mu_F-\mu_C=B(1/0\,.\,486^2-1/0\,.\,656^2)=1{\cdot}91B.$$

$$\therefore \qquad \frac{B}{\mu_D-1}=\frac{\mu_F-\mu_C}{\mu_D-1}\times\frac{1}{1{\cdot}91}=\frac{\tilde{\omega}}{1{\cdot}91} \qquad \text{(15)}$$

where $\tilde{\omega}$ is the dispersive power of the compensator plate, and may be taken as 0·015 for crown glass.

Again, if we define β by the identity

$$\beta\equiv[(\mu_2-\mu_1)_F-(\mu_2-\mu_1)_C]/(\mu_2-\mu_1)_D$$

we have, since $(\mu_2)_F-(\mu_2)_C=1{\cdot}91B_2,$

and $(\mu_1)_F-(\mu_1)_C=1{\cdot}91B_1,$

therefore, $(\mu_2-\mu_1)_F-(\mu_2-\mu_1)_C=1{\cdot}91(B_2-B_1),$

or, $$\frac{(\mu_2-\mu_1)_F-(\mu_2-\mu_1)_C}{\mu_2-\mu_1}=\frac{1{\cdot}91(B_2-B_1)}{\mu_2-\mu_1},$$

that is, $$(B_2-B_1)/(\mu_2-\mu_1)=\beta/1{\cdot}91. \qquad \text{(16)}$$

Introducing the values given by equations (15) and (16) into equation (14), and writing g for the quantity $2\mu(\mu-1)/H(\sqrt{H}-1)$, we have

$$\frac{N_2}{N_1+N_2}=\frac{1+2g\tilde{\omega}/1{\cdot}91\lambda^2}{1+2\beta/1{\cdot}91\lambda^2}$$

$$=\frac{0{\cdot}955\lambda^2+g\tilde{\omega}}{0{\cdot}955\lambda^2+\beta}, \qquad \text{(17)}$$

or, $$\frac{N_1}{N_1+N_2}=\frac{\beta-g\tilde{\omega}}{0{\cdot}955\lambda^2+\beta} \qquad \text{(17a)}$$

Here λ denotes the wavelength of the brightest light in the spectrum of the ½-watt gas-filled Osram lamp used as the source in these experiments. This has been investigated, for example, by Gibson and Tyndall, "Visibility of Radiant Energy" (Bureau of Standards, Sci. Papers, No. 145, v. 19, p. 131, 1923), who find the average luminosity curve for such a source reaches its maximum value at $\lambda=0{\cdot}58\,\mu$.

Again, according to Borgesius (Ann. d. Phys., v. 54, p. 221, 1895), β may be taken as 0·032 for all KCl solutions; 0·036 for NaCl solutions; and 0·042 for all solutions of KNO_3. (*See* also Hallwachs, Ann. d. Phys., v. 47, p. 380, 1892; and Dijken, Zeit. f. ph. Chem., v. 24, p. 81, 1897.)

Also it was found experimentally that for the compensating plates used, $\mu=1{\cdot}511$ and therefore $g=0{\cdot}488$.

Introducing these values into equations (17) and (17*a*), we obtain the results tabulated below :—

TABLE I.

	KCl.	NaCl.	KNO_3.
N_2/N_1	13·30	11·45	9·47
$N_1/(N_1+N_2)$	0·070	0·080	$0{\cdot}095_5$
$N_2/(N_1+N_2)$	0·930	0·920	$0{\cdot}904_5$

In the theoretical, ideal adjustment of the interferometer, which we have been considering, it has been assumed that the achromatic line, defined by the condition $d\varphi/d\lambda=O$, is restored from the displacement x'_1 to zero by a rotation of the compensator plate, and simultaneously the central fringe moves from x'_0 to x''_0—i.e., the recorded travel of the interferometer screw would correspond to a displacement of the central fringe equal to (N_1+N_2) fringe-widths of wavelength λ—the actual displacement of the central fringe by the solutions is N_2 fringe-widths. Hence, $N_2/(N_1+N_2)$ would be the factor by means of which the observations must be corrected for the " wandering " of the achromatic line if the theoretical adjustment of the instrument could be realised.

The expressions deduced in equations (17) and (17*a*), and evaluated in Table I, indicate that in the case of KCl solutions, for example, the achromatic line would move on with respect to the central fringe, one fringe-width for every 13·3 fringes in the displacement of the central fringe ; or, the displacement of the central fringe is 7 per cent. less than that of the achromatic line.

§ 4. PRACTICAL APPLICATION OF THE THEORY.

In the practical application of the theoretical expressions given above, however, the conditions mentioned will not, in general, be fulfilled ; for the achromatic line does not necessarily coincide in position with a fringe, and in compensating for the displacement of the fringes, we cannot restore the achromatic *line* to its undisplaced position, but only the nearest *fringe*, the achromatic fringe.

The modifications necessitated by this fact can be determined in the manner described below :—

N_1 is the number of fringe-widths in the final displacement of the central fringe, when the achromatic *line* is supposed to be restored to the zero position by a rotation of the compensator plate. This in general will not be an integral number.

But in experimental work, as the achromatic *fringe* is substituted for the achromatic line, it necessarily follows that there is always an integral number (say N'_1) of fringes in the final displacement of the central fringe when the compensator plate has been so rotated that the displacement of the achromatic *fringe* is zero.

Now it was found experimentally, by illuminating the slit S_0 (Figs. 1 and 2) by the appropriate source, instead of by white light, that a movement of the screw S (Fig. 2) through 0·00236 inch rotated the compensating plate C through such an angle that the green mercury fringes ($\lambda=0{\cdot}5461$ microns) were displaced through one fringe width. Similarly the movement of the screw was found to be 0·00255 inch for the fringes in Na light ($\lambda=0{\cdot}5893$ microns). In both cases the movement of the screw is almost exactly proportional to the wavelength. By interpolation we deduce the screw-movement necessary to displace the fringes in white light through one fringe space is 0·00250 inch. This method of screw calibration is much more accurate than attempting to estimate the movement in white light directly, since so much sharper fringes can be observed over so much greater a range of movement of the screw when monochromatic light is employed.

Therefore, if r is the final reading of the interferometer screw when the achromatic fringe has been restored to its undisplaced position, and r_0 is the zero reading—i.e., the reading of the screw when the same solution fills both parts of the cell, and the two compensating plates are exactly parallel, so that the achromatic fringes coincide in the upper and lower parts of the field of view of the eyepiece, then

$(r-r_0)/(0{\cdot}0025)$=number of fringes in the total movement of the central fringe, during the process of "compensation"—i.e., $(r-r_0)/(0{\cdot}0025)=N'_1+N_2$ . . . . (18)

Now N_1 and N'_1 can be found as follows :—

As an approximation, take $N_1=N'_1$, and write

$$N_1 \doteqdot [N_1/(N_1+N_2)](N'_1+N_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

The first factor is given by equation (17*a*), and its values have been included in Table I; the second factor is obtained by equation (18), directly from the experimental observations, and thus the approximate value of N_1 is given by equation (19).

As the greatest displacement of the achromatic fringe from the achromatic line is half a fringe-width, it follows that this value of N_1 will also be correct to within this amount, and therefore N'_1 will be found as the nearest integer to N_1.

This expresses the fact that the shift of the achromatic fringe is not continuous but can only take place in steps of one fringe.

Now, from equation (18) we have

$$[(r-r_0)/0{\cdot}0025]-N'_1=N_2,$$

or, from equation (12), since in the apparatus used $E=1$ cm.

$$\{[(r-r_0)/0{\cdot}0025]-N'_1\}\times 0{\cdot}000058=\mu_2-\mu_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

where N'_1 is the nearest integer to

$$[(r-r_0)/0{\cdot}0025][(\beta-g\varpi)/(0{\cdot}955\lambda^2+\beta)]$$

In order to demonstrate the magnitude of the quantities involved, some typical figures are given in Table II.

TABLE II.

Salt.	$r-r_0$	$N'_1+N_2=$ $(r-r_0)/0{\cdot}0025$	N_1 1st approx.	N'_1	N_2
NaCl	0·1496	59·84	4·80	5·00	54·84
	0·1245	49·80	4·00	4·00	45·80
	0·1495	59·80	4·80	5·00	54·80
	0·1350	54·00	4·34	4·00	50·00
	0·0993	39·72	3·19	3·00	36·72
	0·1035	41·40	3·32	3·00	38·40
K Cl	0·1177	47·08	3·29	3·00	44·08
	0·0963	38·52	2·69	3·00	35·52
	0·1154	46·16	3·23	3·00	43·16
	0·1038	41·52	2·90	3·00	38·52
	0·1542	61·68	4·31	4·00	57·68
	0·0798	31·92	2·23	2·00	29·92
KNO_3	0·0883	35·32	3·37	3·00	32·32
	0·1095	43·80	4·18	4·00	39·80
	*0·1440	57·60	5·50	5·50	52·10
	0·1316	52·64	5·02	5·00	47·64
	0·1382	55·28	5·27	5·00	50·28
	0·1113	44·52	4·25	4·00	40·52

* This reading is the mean of settings of two adjacent fringes of apparently equal "achromatism," on the zero of the eyepiece.

The author is indebted to Mr. J. Guild for this method of avoiding the error arising from want of coincidence of the achromatic line with the achromatic fringe.

From equation (20) we obtain the small difference in Refractive Index $(\mu_2-\mu_1)$ for two solutions of known small difference in concentration (n_2-n_1), for the brightest light in the source used—i.e., the value of $d\mu/dn$ for $\lambda=0{\cdot}58$ micron.

In the diffusion experiments to which these results are to be applied, it has been found useless to employ white light for the source, for the reason mentioned in Proc. Phys. Soc., v. 36, p. 322, 1924, and green mercury light ($\lambda=0{\cdot}5461$ micron) was used as the source. It is therefore necessary to make an estimate of the value of $d\mu/dn$ for $\lambda=0{\cdot}5461$ micron.

§ 5. CORRECTION FOR THE GREEN MERCURY LINE.

To correct the value of $(d\mu/dn)$ found by the interferometer for $\lambda=0{\cdot}58$ to its value for the green Hg line, $\lambda=0{\cdot}5461$, assume

$$\mu=A+B\lambda^{-2}$$

i.e. $$\left(\frac{d\mu}{dn}\right)_{\lambda_1}=\left(\frac{dA}{dn}\right)_{\lambda_1}+\lambda^{-2}{}_1\left(\frac{dB}{dn}\right)_{\lambda_1} \text{ for a wave-length } \lambda_1$$

and $$\left(\frac{d\mu}{dn}\right)_{\lambda_2}=\left(\frac{dA}{dn}\right)_{\lambda_2}+\lambda^{-2}{}_2\left(\frac{dB}{dn}\right)_{\lambda_2} \text{ for another wave-length } \lambda_2,$$

where A and B are functions of n, but not of λ.

From published values of μ and λ for solutions of sodium chloride of different concentrations it will be found that dA/dn is an approximate constant, having the value, independent of λ, of about 0·1541, and similarly dB/dn has an approximately constant value, independent of λ, equal to about $0{\cdot}2961\times10^{-10}$. Although it is not claimed that the accuracy of these figures is very high, yet the correction involved is so small that they are quite sufficient for our purpose. This is on account of the fact that the green mercury line is situated so near the brightest part of the spectrum.

Hence $(d\mu/dn)_{\cdot58}=0{\cdot}1541+[1/(0{\cdot}000058)^2]\ 0{\cdot}2961\times10^{-10}$

$=0{\cdot}1628$ approximately.

Similarly $(d\mu/dn)_{\cdot5461}=0{\cdot}1541+[1/(0{\cdot}00005461)^2]\ 0{\cdot}2961\times10^{-10}$.

The difference, $(d\mu/dn)_{\cdot5461}-(d\mu/dn)_{\cdot58}$

$=0{\cdot}2961\times10^{-10}[1/(0{\cdot}00005461)^2-1/(0{\cdot}000058)^2]$

$=0{\cdot}001127.$

i.e., the value of $(d\mu/dn)$ for green mercury light is greater than that for wave length 0·58 micron by 0·00113 in 0·1628—that is, by 0·7 of one per cent.

Corrections of the same order of magnitude are found in the cases of KCl and KNO_3 solutions, so that the values of $(\mu_2-\mu_1)$ given by equation (20) must be increased by 0·7 per cent. to correct for the wavelength of the light employed.

§ 6. RESULTS FOR $(d\mu/dn)$.

It has been shown that if r_0 is the zero reading of the compensator screw, and r is its reading when the interferometer cell contains two solutions of slightly different concentrations n_1 and n_2, then the equation

$$\{[(r-r_0)/0{\cdot}0025]-N'_1\}\,0{\cdot}000058\times 1{\cdot}007=d\mu \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

expresses the small difference in their refractive index, where corrections have been introduced to amend the observations of the interferometer for (1) difference in dispersion of the media, and (2) the wavelength of the light employed.

An application of equation (21) enables the last column of Table III to be evaluated.

TABLE III.

Salt.	Concentrations. n_1	n_2	Mean. n	$r-r_0$	$d\mu/dn$
NaCl	0·2500	0·2273	0·239	0·1496	0·1409
	0·1667	0·1488	0·158	0·1245	0·1497
	0·1250	0·1042	0·115	0·1495	0·1537
	0·0625	0·0446	0·053	0·1350	0·1635
	0·0250	0·0125	0·019	0·0993	0·1716
	0·0125	0·0000	0·006	0·1035	0·1794
KCl...........	0·2500	0·2273	0·239	0·1177	0·1132
	0·1667	0·1488	0·158	0·0963	0·1161
	0·1250	0·1042	0·115	0·1154	0·1210
	0·0625	0·0446	0·053	0·1038	0·1260
	0·0250	0·0000	0·0125	0·1542	0·1348
	0·0125	0·0000	0·006	0·0798	0·1397
KNO_3.........	0·2500	0·2273	0·239	0·0883	0·0830
	0·1667	0·1389	0·153	0·1095	0·0836
	0·1250	0·0893	0·107	0·1440	0·0852
	0·0625	0·0312	0·047	0·1316	0·0890
	0·0312	0·0000	0·0156	0·1382	0·0937
	0·0250	0·0000	0·0125	0·1113	0·0948

These values of $(d\mu/dn)$ are slightly different from those given in Proc. Phys. Soc., v. 36, p. 313, 1924, owing to a more correct value being adopted for the dominant wavelength of the light emitted by the Osram lamp.

All the experiments were conducted at temperatures between 18° and 19°C., and the solutions used were prepared by the use of standardised and calibrated pipettes of volumes 50, 25, 10 and 5 c.c.

The figures tabulated above in Table III are the mean results of a number of experiments which only differ from each other by quite small amounts, and they are plotted graphically in the curves of Fig. 6.

§ 7. DISCUSSION OF RESULTS.

In the application of these results to the author's experiments on diffusion, referred to in § 1, the following considerations should be borne in mind :—

(1) The difference $(d\mu)$ for two solutions of known difference in concentration

(dn) has been measured directly, instead of depending on the difference of two values of μ, each of which must be separately measured to a very high degree of accuracy.

(2) The interferometer measurements have been made in the same thermostat room, and at the same temperature, as the observations on diffusion to which they are to be applied.

(3) The correction for difference between the wavelength of the light used in the interferometer ($\lambda=5800$ Å) and that employed in the diffusion experiments ($\lambda=5461$ Å) is not large.

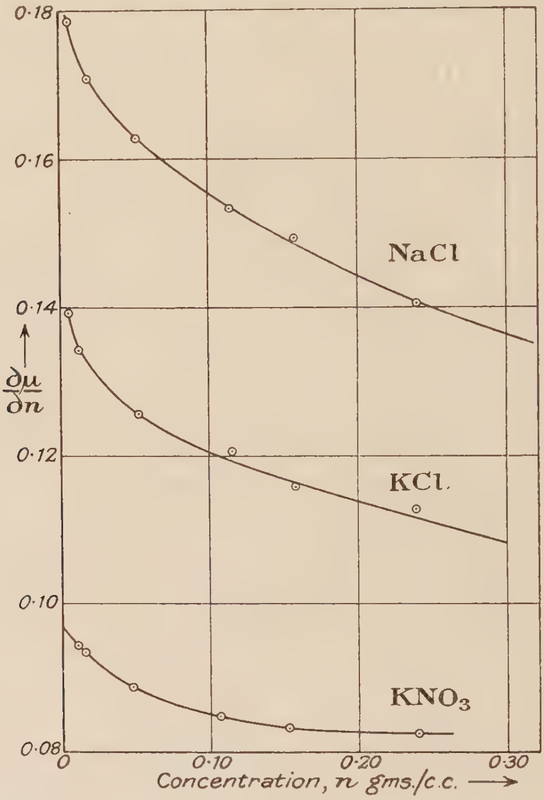


FIG. 6.

Accuracy.

The object with which this Paper is submitted is to describe a series of experiments constituting a subsidiary investigation in connexion with the optical method of studying diffusion recently described by the author before this Society, in which an estimate is required of the difference in refractive index of two solutions to the order of accuracy at present requisite in experimental work in diffusion—say, 2 per cent. or so.

The mean values of ($d\mu/dn$) given above in Table III are considered to be accurate to well within this limit, and if necessary this accuracy could be considerably increased, for almost all the error involved is due to the method which has usually been adopted in the preparation of the solutions—a method employed on account of its speed, its simplicity, and its reasonable accuracy—viz., the difference (*dn*) in concentration between the two solutions whose difference ($d\mu$) in refractive index is to be measured has usually been produced by the addition of a known volume of water to a known volume of the solution to be studied by means of calibrated pipettes. By the substitution of a chemical analysis of the solutions employed in the interferometer cell for the use of the pipettes, the error involved could be reduced to a considerable extent.

That a greater accuracy may now be thought to be not only desirable (in view of the interest connected with the diffusivity of dilute solutions), but also possible, indicates that a considerable advance has been made in measurements on diffusion; but, as the author has previously stated, he feels that the diluter solutions should be the subject of further study, in order that the conclusions suggested by his recent work may be confirmed, and he intends to proceed with this investigation as soon as possible.

The work has been carried out in the Physical Laboratories of Birkbeck College, and the author is glad to express his indebtedness to Prof. A. Griffiths for his continued interest.

DISCUSSION.

Mr. J. GUILD: The author has called attention to a particular case of a general result in connexion with interferometer apparatus, namely, that complete compensation is only obtained in an interferometer if the materials and dimensions of the optical elements in each beam are identical. The Paper is of theoretical interest, but for practical purposes white light should not be used. It is possible, starting with an approximate knowledge of the quantity to be measured, to obtain values as accurate as may be required using monochromatic light. It is only necessary to limit the path difference introduced so that the approximate initial knowledge enables the number of fringes displaced to be calculated with an uncertainty of less than half a fringe. This gives the integral number of fringes in the displacement, the fraction being obtained from the actual setting. If desired, the more accurate value now obtained may be used as the starting point for another determination, using a correspondingly greater path difference. In this way a series of observations may be made each giving an accuracy of from five to ten times that of the preceding.

Mr. T. SMITH: The author's investigation shows how complicated such problems become when white light is used; I should have thought monochromatic light preferable in this case. For a rigorous treatment it would be necessary to consider a further correction, arising out of the fact that the dominant wavelengths may be different in the two fringes which have to be compared, but possibly such a correction would be negligible for the purpose under consideration. For strict rigor it would also be necessary to consider dispersion in the eyepiece system.

AUTHOR's reply: I must express my gratitude to Mr. Guild for his suggestions as to the use of monochromatic light in the measurements under consideration. With regard to Mr. T. Smith's remarks, I have not allowed for a possible variation in the dominant wavelength in the two sets of fringes, but imagine this would produce only a very small modification in the results, of little importance in their application to diffusion.

XI.—A STUDY OF THE PRODUCTION OF "FLASHING" IN AIR ELECTRIC DISCHARGE TUBES.

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ABSTRACT.

The Paper deals with an extension of the study of "flashing" in discharge tubes* containing air instead of the usual mixture of neon and helium.

The volt-ampère characteristics for the air discharge tubes are shown to be of the general form,

$$i=k(V-M),$$

where i is the current through the tube, k the conductance of the tube, V the voltage across the electrodes, and M a constant, theoretically of the value of the kathode fall of potential V_A.

The relation for the time of "flashing" T is given theoretically by

$$T=CR\,.\log_e\frac{E-V_B}{E-V_C}+C\Delta\,.\log_e\frac{V_C-V_A}{V_B-V_A}$$

where C is the capacity in microfarads across the tube, R is the resistance in megohms, in series with the tube, E, V_C, V_B, the charging, upper critical, and lower critical voltages respectively, and Δ a constant.

This relation is generally confirmed by experiment.

It is shown that, as in the case of the "osglim" lamp, there is a critical resistance R_c below which no "flashing" is possible, given by

$$R_c=\frac{E-V_B}{k(V_B-V_A)}.$$

INTRODUCTION.

THE following work is a continuation of the study of "flashing" of the neon tube,† which contains a mixture of the rare monatomic gases neon and helium; it was of considerable interest, therefore, to investigate the phenomenon in a diatomic molecular mixture like air, and to see if the same laws were valid.

It is to be observed that the theory put forward previously is theoretically applicable solely to discharge tubes in which the positive column is absent (so that no repetition of the dark space phenomena occur); thus the electrodes of the tube must not be more than some few mm.s apart.

Experiment shows that in the case of air discharge tubes of very varied types the phenomenon of "flashes" may be readily obtained. The range of pressures for which "flashes" are possible is very wide (0·35 to 40 mm. in these experiments) for a voltage range of some 400 up to 1,000 volts. Furthermore, "flashes" of almost any period can be obtained.

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* First observed by Pearson and Anson, in the case of the neon-filled "osglim" lamp (Proc. Phys. Soc., Vol. 34, p. 204.)

† Pearson and Anson, loc. cit. Taylor and Clarkson, Journ. Scien. Instrs., Vol. 1, p. 173; also Proc. Phys. Soc., Vol. 36, Pt. 4, p. 269.

EXAMINATION OF THE UPPER AND LOWER CRITICAL VOLTAGES.

It was found that for the various discharge tubes, under different given conditions of temperature and pressure, a definite potential across the electrodes—the "sparking" potential or upper critical voltage V_C—was required to initiate discharge. V_C was definite and repeatable to within one or two volts, provided the discharge tube remained under identical conditions.

It is important that V_C should be measured under conditions similar to those which will exist during experiments (for method, see later).

Graph (1) of Fig. (1) shows the variation of V_C with pressure; the curve is of similar form to that obtained for the "sparking" potential of gases. There is a flat minimum in the neighbourhood of 2·5 mm., and at greater pressures than 3·5 mm.s V_C becomes a linear function of the pressure.

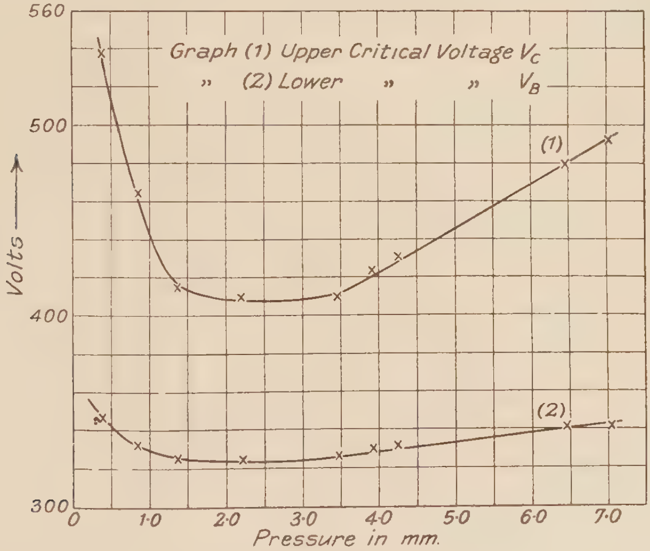

FIG. 1.—GRAPHS SHOWING THE VARIATION OF THE UPPER AND LOWER CRITICAL VOLTAGES (FOR AIR), WITH PRESSURE.

For air, as for other gases, there is a lower critical voltage V_B below which no discharge may be maintained. V_B is of constant and repeatable value for a definite pressure, under given conditions. Graph (2) of Fig. (1) shows the variation of V_B with pressure. There is a flat minimum at the same pressure as the minimum for V_C, and at higher pressures V_B increases linearly, with a small slope compared with that for V_C.

EXPERIMENTAL METHOD FOR THE DETERMINATION OF V_C AND V_B.

Fig. 2 gives the experimental arrangement used for the determination of V_B and V_C. With suitable values of the capacity C and the resistance R, "flashing" occurs. E, the charging voltage, is gradually lowered in value (R being reduced at the same time to decrease the time period of "flashes") till no further "flashing" occurs; the value of E is then that of the upper critical voltage. It is important that V_C should be measured under conditions similar to those which will exist during experiments.

E is then raised so as to be greater than V_C, so that "flashing" occurs, R is reduced to the critical value R_c (*see* theoretical discussion), and the discharge becomes continuous. The voltage across the tube at this adjustment is V_B* (an electrostatic voltmeter is employed for the measurement of the voltage).

THE VOLT-AMPÈRE CHARACTERISTICS OF AIR DISCHARGE TUBES.

One of the present writers obtained theoretically, on certain assumptions, a current-voltage law of the following form† (no resistance being in series with the discharge tube) :—

$$i = k(V - V_A) \quad \ldots \ldots \ldots \ldots \quad (1)$$

where i is the current through the tube, V the voltage across the electrodes, V_A the kathode fall of potential (approximately), and k is a constant depending upon the area of the kathode utilised during the discharge, and may be termed the conductance of the tube.

The above relation agrees substantially with that obtained for the neon lamp,‡ and it should be applicable to any discharge tube in which the positive column is absent.

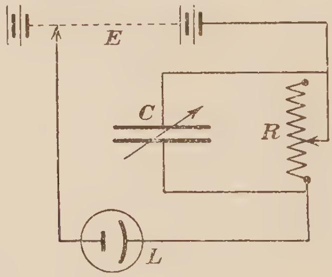

FIG. 2.

In the present experiments the characteristics for the air discharge tubes were determined, between the upper and lower critical voltages. Although the method was by no means free from objections, the volt-ampère characteristics (Fig. 3) for different pressures were approximately of the linear form,

$$i = k(V - M) \quad \ldots \ldots \ldots \ldots \quad (2)$$

M, the intercept on the voltage axis, which should be of the value V_A, is almost constant for different pressures and is of the order of magnitude expected.

In the case of many tubes the results were, however, very unsatisfactory. Provided, however, that M is not interpreted literally as V_A, the kathode fall of potential, but rather as a constant of meaning at present undetermined, the results of observation agree substantially with the theoretical relationship of equation (1).

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* Taylor and Clarkson, Proc. Phys. Soc., Vol. 36, Part 4, pp. 273, 276.

† Journ. Scient. Instrs., T. and C., loc. cit.

‡ P. and A., loc. cit.

EXPERIMENTAL METHOD FOR THE DETERMINATION OF THE VOLT-AMPERE CHARACTERISTICS.

Fig. 4 indicates the experimental arrangement for the determination of these characteristics. A potentiometer was used so that the voltage across the tube could be varied. *A* was a milliammeter for measurement of the current through the tube, E_v, an electrostatic voltmeter for determining the potential across the tube terminals, and *R* was a variable resistance.

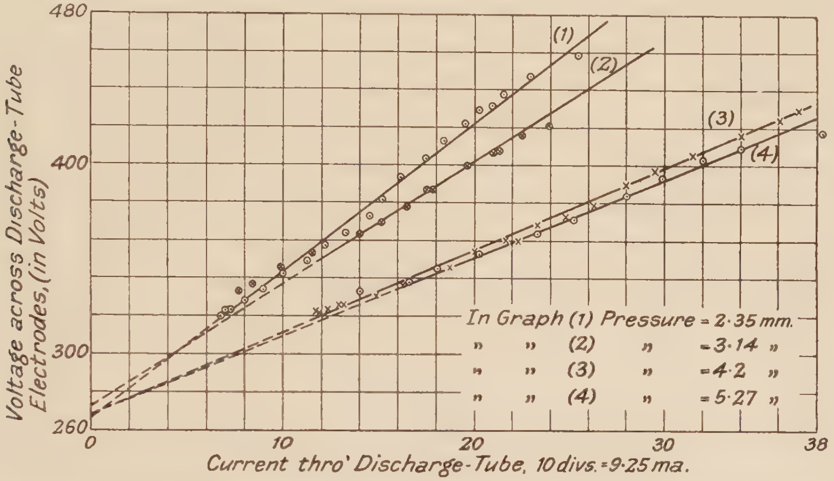

FIG. 3.—GRAPHS OF THE VOLT-AMPERE CHARACTERISTICS FOR AIR AT DIFFERENT PRESSURES.

A preliminary experiment was made (with *R* adjusted to be several thousand ohms) to ascertain the approximate values of V_B and V_C.

With *R* of zero value, the voltage *E* was adjusted to be just less than V_C, and was then stepped up gradually until the discharge started. As soon as the current through the discharge tube had fallen to a steady value, readings of the voltage and current were obtained simultaneously. The voltage was then progressively diminished, whilst the values of the current were obtained, until it had diminished to the lower critical voltage, when the current dropped abruptly to zero.

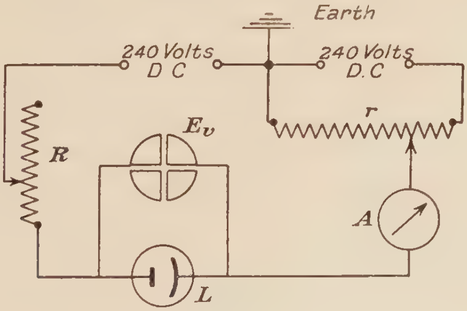

FIG. 4.

It is well known that the value of the current through a discharge tube under these conditions of running on a steady voltage, falls off almost exponentially from its initial value, with time, to a definite assymptotic value.* This would appear to vitiate the value of the above results, for it is evidently the instantaneous value of the current, during the short luminous period, that must be taken into account in the "flashing" phenomena.

It was found, however, provided the current was not more than some 10 or 15 milliamperes, that the initial values of the current were not greatly in excess of the final values, and that the results were repeatable to within a few per cent. When the discharge became sufficiently intense to produce great local heating, erratic results were obtained.

On the whole, it is reasonable to assume that the magnitudes only, and not the form of the volt-ampère characteristics, will differ in the two cases of the final and the instantaneous values.

THE THEORY OF "FLASHING."

There are two positions for the capacity for which "flashing" may be obtained. The first position is where the capacity is across the tube terminals, and the second position is where the capacity is across the resistance (Fig. 2).

The theory of the first position has been put forward in previous Papers,† the time period being given by the relation

$$T=CR\,.\,\log_e\frac{E-V_B}{E-V_C}+\frac{CR}{1+kR}\,.\,\log_e\frac{V_C-\dfrac{E+kRV_A}{I+kR}}{V_B-\dfrac{E+kRV_A}{1+kR}}\quad\ldots\ldots\quad(3)$$

where C is the capacity across the tube in microfarads, R is the series resistance in megohms, and E is the charging voltage. (Other terms have the same meaning as before.)

The second position of the condenser is characterised by a time period relation of exactly the same form as that given by equation (3).

The condensers available made it more convenient, in the present experiments, to use the second position of the capacity. When (as is usually the case) kR is large compared with unity, the relation of equation (3) may be approximated to the more simple form‡

$$T=CR\,.\,\log_e\frac{E-V_B}{E-V_C}+\frac{C}{k}\,.\,\log_e\frac{V_C-V_A}{V_B-V_A}\quad\ldots\ldots\ldots\ldots\quad(4)$$

The time period should, therefore, be directly proportional to C, over a wide range of capacities; further, since the first term of equation (3) is very large compared with the second, T should be a linear function of R, over a wide range of resistances. When, however, R becomes small, the product kR progressively diminishes with R, and the second term of equation (3) increases rapidly, so that the linear relation is departed from, T being of greater value than that given by a linear form.

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* P. and A., loc. cit. Shaxby and Evans, Proc. Phys. Soc., Vol. 36, Pt. 4, p. 253.

† P. and A., loc. cit. T. and C., Journ. Scient. Instrs., loc. cit.

‡ T. and C., Journ. Scient. Instrs., loc. cit.

It is further evident if CR is fixed, that T should be a linear function of the logarithm term

$$\log_e \frac{E-V_B}{E-V_C}$$

THE CRITICAL RESISTANCE R_c FOR "FLASHING."

It was shown previously that there is a limiting value for R, called the critical resistance R_c, below which no "flashing" is possible, given by the relation,*

$$R_c=\frac{E-V_B}{k(V_B-V_A)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The second capacity position is characterised by a critical resistance, given by an exactly similar relation. R_c should therefore be a linear function of E, intercepting the axis of voltage, at a value V_B.

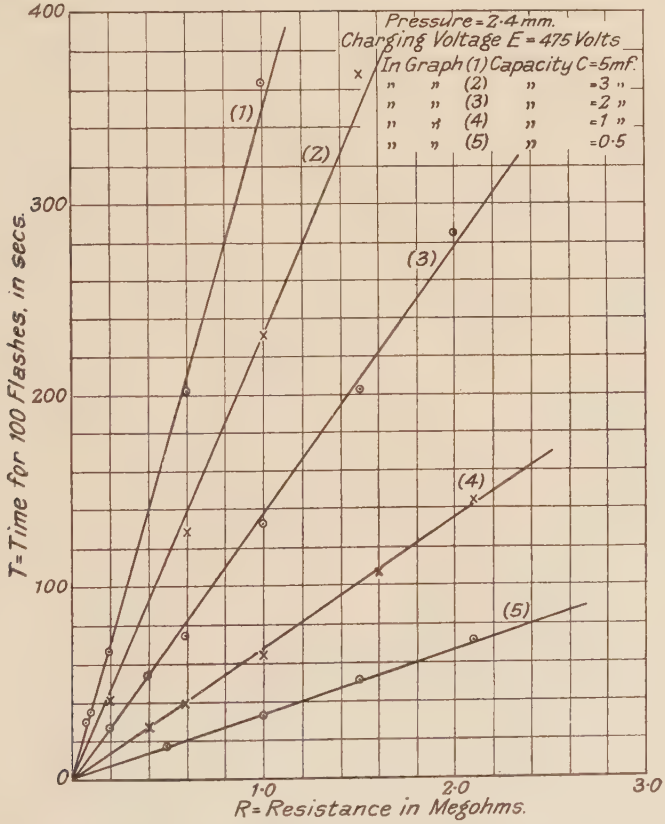

FIG. 5.—GRAPHS SHOWING RELATIONSHIP BETWEEN THE TIME OF FLASHING T AND THE RESISTANCE R FOR DIFFERENT CAPACITIES.

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* T. and C., Proc. Phys. Soc., loc. cit.

EXPERIMENTAL EXAMINATION OF THE RELATIONS OF THE TIME PERIOD OF "FLASH" TO RESISTANCE AND CAPACITY.

The method of experiment was exactly similar to that adopted in the experiments of previous Papers, except that all the tubes used were attached to a Toepler pump in order that the pressure of the air in them could be varied. The pressure was measured by means of a McLeod gauge.

In order to ensure dryness for the air used, phosphorus pentoxide drying-tubes were attached to the discharge tube apparatus.

The graphs of Figs. 5 and 6 show the relationship between the period of "flashing" T, and the resistance R for different capacities, and at two different pressures. (The examples are taken from a numerous series, at different voltages, but they were all of exactly the same form so that inclusion of the whole series is unnecessary.) The discharge tube was that used in the volt-ampere characteristic, and the critical voltage determinations, and consisted of an ordinary "I" pattern

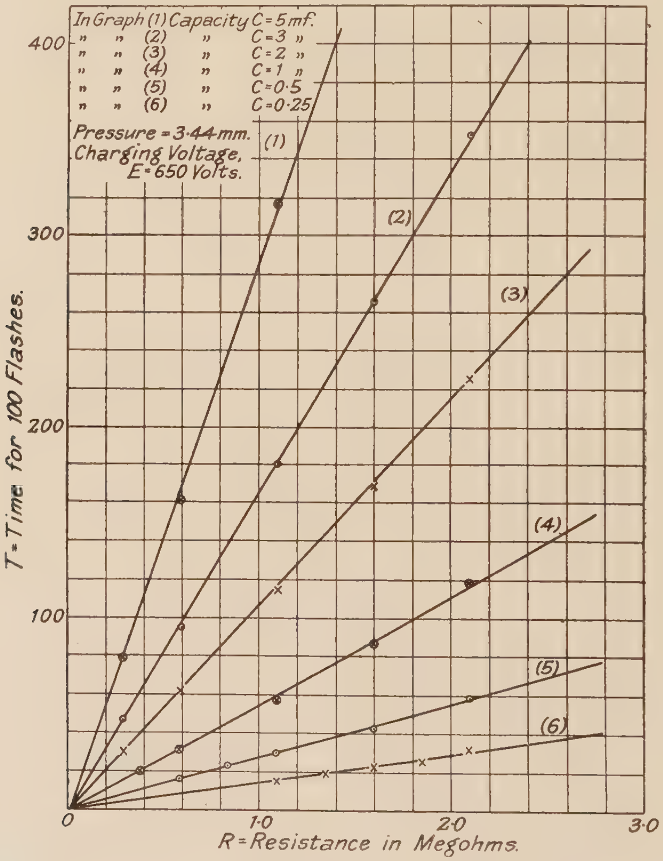

FIG. 6.—GRAPHS SHOWING RELATIONSHIP BETWEEN THE TIME OF FLASHING T AND THE RESISTANCE R FOR DIFFERENT CAPACITIES.

"osglim" lamp which was sealed on to the apparatus for exhausting of air. The dimensions were approximately:—

Kathode plate (iron) 3·5 by 0·5 cm. Anode cylinder 1·5 by 0·2 cm. The cylinder was parallel to the kathode (along same axis) and the minimum distance between them was 0·15 cm.

The graphs are all linear functions, except when *R* is small and near the critical resistance value; this relation holds over a wide range of pressures. Further, the intercepts on the axis of resistance are small, in accordance with the relation of equation (4).

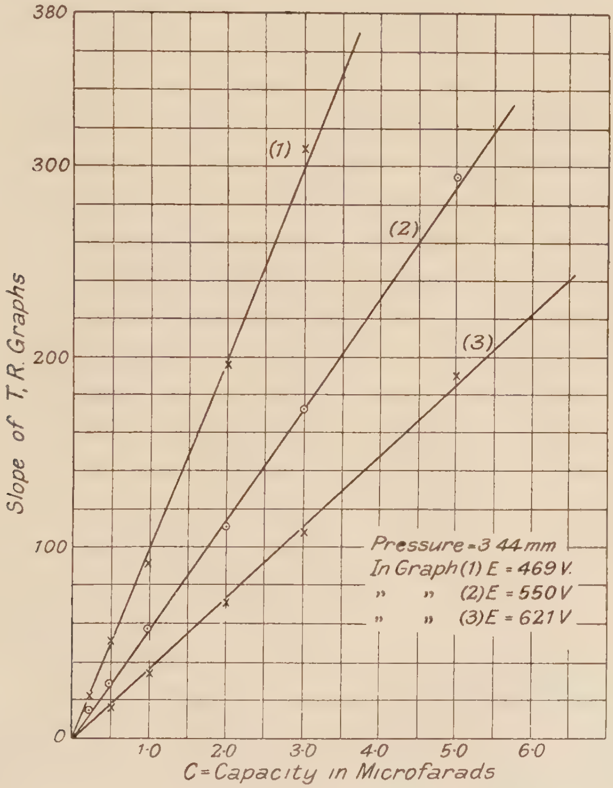

FIG. 7.—GRAPHS SHOWING RELATIONSHIP BETWEEN THE SLOPES OF THE *T*, *R* GRAPHS AND CAPACITIES FOR DIFFERENT VOLTAGES *E*.

The graphs of Fig. 7 are examples of the type of relationship between the slope of the *T*, *R* graphs and the capacity *C*. They are linear, and intercept the axis of capacity at points very near the origin, in accordance with the theory.

The graphs of Fig. 8, taken for a discharge tube with copper disc electrodes (distance apart of electrodes 0·5 mm., diameter of electrodes 3·0 cm.) show the curving off of the *T*, *R* graphs as *R* decreases, for various pressures.

In the case, however, of the variation of *T* with the logarithm factor, an anomaly arises. T was found approximately to vary linearly with this factor (*see* graphs of Fig. 9), but the relation was of the form

$$T=A\left(\log_e \frac{E-V_B}{E-V_C}-B\right) \quad \ldots\ldots\ldots\ldots \quad (5)$$

where A and B are constants.

B is of small magnitude, but A, the slope of the T, log. factor graphs, was always found to be greatly in excess of the theoretical value CR. (In the graphs of Fig. 9, since the ordinates are the time periods $100T$, for C and R both unity, this time period should be very approximately equal to $100 \,.\, \log_e \frac{E-V_B}{E-V_C}\Big)$.

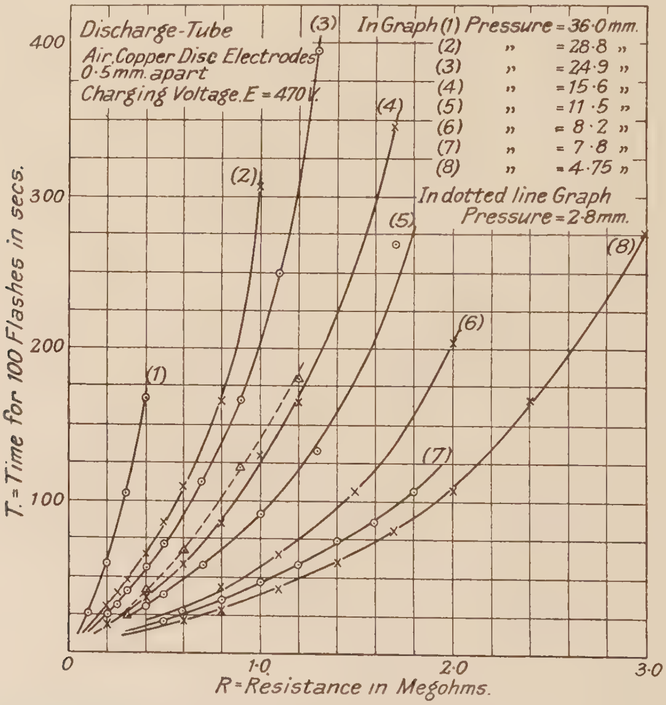

FIG. 8.—GRAPHS SHOWING RELATIONSHIP BETWEEN T, THE TIME OF "FLASHING," AND THE RESISTANCE R FOR DIFFERENT PRESSURES.

The discrepancy remains very great even though the duration of the luminous period of "flashing" is taken into consideration. In a few cases the time period as calculated from equation (4) agreed very closely with the values obtained experimentally; these however, were the exceptions.

The anomaly may be bound up in any variable factors of equation (4), or in a combination of them, and it has not been found possible, up to the present, to elucidate it.

Experiments with various types of air discharge tubes showed definitely that

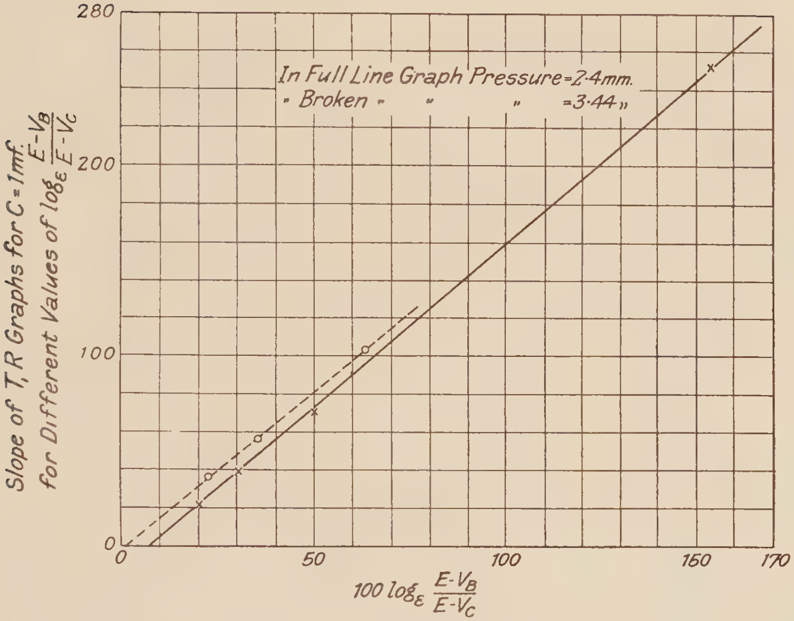

Fig. 9.—Graphs showing the Relationship between the Slope of the T, R Graphs for $C=1$ microfarad (for Different Values of E) and $\log_e \frac{E-V_B}{E-V_C}$.

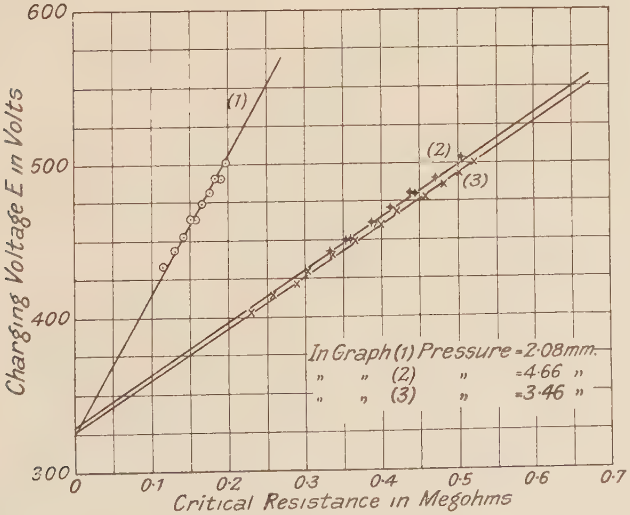

Fig. 10.—Graphs showing Relationship between R_c, the Critical Resistance, and E the Charging Voltage.

there was always a critical resistance for "flashing." In many cases the magnitude of this critical resistance was not more than a few thousand ohms.

It was also ascertained that at the critical resistance, the voltage across the tube fell to the lower critical value V_B, and that R_c was a linear function of the charging voltage E.

The graphs of Fig. 10 show the form of the R_c, E graphs, for the capacity across the resistance, in the case of a wireless valve improvised as a discharge tube. They are straight lines, intercepting the voltage axis approximately in accordance with the relation of equation (5). The results obtained with the majority of the tubes were neither very definite nor conclusive, and it is proposed to investigate the problem more fully.

The authors beg to acknowledge their indebtedness to Prof. G. W. Todd of Armstrong College, under whose supervision the experiments were carried out.

DISCUSSION.

Dr. D. **Owen**: While the authors have made interesting observations on the existence of the phenomenon of flashing in air discharge tubes, and have pointed out the existence of a critical resistance, it is necessary to remark that their observations were made under limited conditions. For instance, the rate of flashing seems not to have exceeded about 5 per second, and the capacities used were in every case large. The assumption also of a straight volt-ampere characteristic is only very partially true.

Mr. E. J. **Evans**: The phenomenon of flashing in discharge tubes was shown by me before the Society in February, 1923. I would like to ask the authors what was their source of potential in the experiments, and how they measured the frequency of the flashes. In Figs. 6 and 8 the authors show that an alteration of electrodes produced an anomalous result. I have observed in my own work using discharge tubes of different lengths and shapes that the results obtained are not in agreement with those given in this Paper, e.g., I obtain intermittence without having any capacity in parallel with either the tube or resistance, while the resistance in series is decidedly lower than the critical values given. I do not get the linear relationship for the volt-ampère characteristic. I find also that at high gas pressure the discharge is intermittent and that the rate of intermittence increases with a decrease of pressure. At a certain critical pressure the luminous period increases and on further reducing the pressure the discharge becomes apparently continuous. Still further reduction of pressure produces intermittence again, the frequency now being low.

Prof. E. V. **Appleton** and Mr. **Emeléus** (communicated): The results described in the above Paper are in agreement with those obtained by Appleton, Emeléus and Barnett (Proc. Camb. Phil. Soc., XXII, p. 434, 1924), when studying the action of Rutherford and Geiger's apparatus for counting α and β-particles electrically. It was found that the transient discharge between a point and a plane, initiated by an ionizing particle in air at reduced pressure, was one unit of the sustained periodic discharge which set in at sufficiently high voltages or low pressures, and that the oscillations were very similar to the "flashing" of a neon lamp. Linear relations between capacity and time period and the characteristic variation of period with resistance were described together with oscillographic determinations of the wave-forms. The counter may be set up with the condenser in either of the two positions. The interesting result that the lower critical voltage decreases when the capacity is sufficiently reduced is in concord with our ballistic measurements. For capacities below a certain amount the quantity of electricity circulating for each discharge actually increased as the capacity decreased. This was attributed at the time to a change in the conductance K, but it is now clear that it is the change in the lower critical voltage which brings about this result. That the latter view is the correct one is verified by direct measurements of the fall of potential across the counter for single discharges made by Geiger (Zeit. für Phys., 27, 7, 1924). Our results have also shown that the effective capacity of the discharge tube and condenser is larger than that calculated from the geometry of the system, and that "flashing" can take place when the positive column is present.

AUTHORS' reply (communicated): The work was necessarily restricted owing to lack of suitable sources of voltage and capacities. The linearity or non-linearity of the volt-ampère characteristics does not gravely affect the general phenomena of "flashing," except at points in the vicinity of the critical resistance. This is seen to be true from a consideration of equations (3) and (4), the first term, as a rule, being of large magnitude compared with the second, and also independent of the form of the characteristic.

In the Paper the portion on the critical resistance is merely illustrative, and as indicated in the addendum a more exact study is given elsewhere (Taylor and Clarkson, Phil. Mag., Vol. 49, No. 290, p. 336). It is shown in the Paper referred to that the linearity of the E, R_c graphs is independent of the form of the volt-ampère characteristics.

In reply to Mr. E. J. Evans, we regret that we are not in a position to discuss the results put forward, since there is no indication of the types of discharge tubes used, the gas employed or the working pressures, nor is it stated what methods were adopted for the observing of successful experimental conditions. We may be permitted, however, to point out that we ourselves have experimented on numerous tubes which gave entirely anomalous and erratic results. Sources of trouble are extremely numerous. Some of the inherent experimental difficulties are described in the Paper referred to (T. and C., Phil. Mag., loc. cit., pp. 338, 345, 353). In addition, the tubes used for the "flashing" relations must have kathodes of suitable size (if linear relations are to be obtained), so that the working conditions are not situate in the range where great change of the critical constants with diminution of capacity occurs. It is obviously impossible to apply the relation of equation (2) to the "flashing" of a tube, if the critical constants are changing rapidly with capacity. (For change of constants under different conditions, see T. and C., Phil. Mag., loc. cit.; Taylor and Stephenson, Journ. Sci. Instrs., Vol. 11, No. 2, p. 50; and Taylor, Clarkson and Stephenson, Journ. Sci. Instrs., Vol. II, No. 5, p. 154.)

Grave departures from linear relations are frequently due to a "lag" in the discharge behind the voltage tending to produce it, owing to insufficiency of ionization within the tube. This may be overcome by the utilisation of an external ionizing agent, such as γ-rays (*see* T. and S., loc. cit.; T., C. and S., loc. cit.). We do not know whether Mr. Evans utilised tubes in which there were positive columns. If this were the case, it is obviously impossible to compare the results with ours, since we confined our attention definitely to tubes fulfilling the conditions laid down in the Paper.

We do not agree that Figs. 6 and 7 show anomalous results. The discharge tube used in obtaining the results of Fig. 7 had a conductance greater than those of the other tubes we employed, and the working pressures were much greater. It is apparent, therefore, that the second term of equation (3) cannot be taken as a constant, especially when it is remembered that R_c is greater for higher pressures, and as this value is approached the second term (which is the luminous period time) becomes greater and greater. Further, we had ascertained by actual experiment that for this particular tube, at a constant pressure and fixed values of R and C, the time period was a linear function of the logarithm term, in accordance with equation (3).

The observation of the production of "flashing" or intermittance in tubes without any capacity in parallel is interesting (the magnitude of the critical resistance has no significance in comparing results, however, since it is a function of the conductance of the tube, and has an indefinite range of possible values). It has been observed previously, however, by Ives, in work on air discharge tubes on the so-called magnetic rays of Righi, by Aston and Watson in the case of rare gas discharge tubes, and by others. In most of the cases where slow "flashing" under these conditions has been observed there has been a positive column, and the conditions of the two cases are scarcely comparable. In the case of high-frequency flashes, if these are the type referred to, the residual capacity of the circuit and discharge tube electrodes is usually sufficiently great to account for them. When the value of the charging voltage E is very near to the upper critical voltage value, it is seen, from a consideration of equation (3), that the phenomena of "flashing" could occur at slow frequencies even with low capacity and resistance values. We suggest that the explanation of Mr. Evans's last results may be deduced from a consideration of equation (3), together with the empirically obtained curves of the variation of the upper and lower critical voltages with the pressure (Fig. 1), and the critical resistance condition for the maintenance of "flashing" in the circuit.

The authors are greatly indebted to Prof. E. V. Appleton and Mr. Emeléus for their communication, and it is hoped to be able to carry out some of the experiments described in their Paper on the particular type of discharge tube that we have used.

XII.—NOTES CONCERNING THE SPRENGEL PUMP.

By J. J. MANLEY, *M.A.*, Research Fellow, Magdalen College, Oxford.

ABSTRACT.

The Paper describes an improved design of the modified Sprengel pump described by the author in the Proceedings of the Physical Society, Vol. 36, p. 291. The walls of the pump are freed from gas skins by electric means, and the shattering of the mercury pellets (which tends to liberate gas) is checked by a special construction of the fall tube.

IN a former Paper* I showed how by an electrical method the interior surfaces of a Sprengel pump and also the inflowing mercury could be freed, the one from the gas-grown skins and the other from adsorbed and condensed gases. It was also shown that in the absence of gases obtainable from the glass walls of the pump and possibly from the mercury, the pump acquired a much greater and in all probability the maximum efficiency attainable.

Recently, other pumps have been constructed ; each of these was given a form differing but slightly from that of the parent pump of my earlier Paper. After experimenting with these, choice was made of that illustrated in Fig. 1. The chambers *A* and *B* of this pump are 20 cm. long and 2 cm. wide ; their upper portions are closely fitted with external electrodes *C*, *D* of tinfoil, 10 cm. long. These electrodes are formed and attached as described in my former communication ; but on account of its greater reliability, Canada balsam, rendered conveniently liquid by adding xylene, was substituted for the alcoholic solution of shellac used in previous cases. Internal electrodes of mercury are provided by fusing platinum wires *w*, *w*, into the lower part of each chamber ; placed thus, the ends of the wires are completely submerged when the pump is fully charged with mercury. The tubes *Q* and *R* are fused, the one to a barometric trap and the other to a Töpler pump ; the trap is placed between the head of the Sprengel pump, and any vessel we may wish to evacuate. This new pump is prepared for use by proceeding in the way already described.*

When the coil *J* is activated by a battery *E* of three accumulators in series, the ensuing glow discharge within the chambers is chiefly concentrated in the spaces between the mercury and the interiors of the tinfoil electrodes ; this discharge erupts the gas-grown skins of the interior surfaces, and possibly some of the gases adsorbed by the mercury. By concurrently using the Töpler pump and coil, the chambers are quickly evacuated. The Sprengel pump is ready for action when a green fluorescent glow appears in the chambers *A*, *B*. From this time onwards the coil and the auxiliary pump are seldom required for more than a few seconds daily. It therefore appears that the mercury used for working the pump carries little or no air with it ; or, if it does so, such air is not readily detached by the glow discharge in vacuo ; hence we conclude that the primary source of the air which under usual conditions, limits the efficiency of a Sprengel pump, must be the internal surfaces of the pump itself.

The bulb *P* contains phosphorus pentoxide purified by ozone.† The oxide is so placed that the chambers *A*, *B* and the mercury within them, are kept dry; the oxide also removes moisture from the Töpler pump, and the connecting tube *R*. The new pump is to be preferred to the original in that it is simpler in form, equally efficient, and also silent in action. In the older pump, the mercury falls from a glass jet at the top of either chamber, on to the surface of the metal beneath, and so produces considerable noise. I conclude by describing two minor improvements.

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* Proc. Phys. Soc., Vol. 36, Part 4, p. 291.

‡ Trans. Chem. Soc., Vol. 121, p. 331.

(*α*) With some pumps there is at times a tendency for the mercury to stick and accumulate in the fall-tube and head; and frequently a difficulty is experienced in re-starting its flow. This defect is remedied by making the head-piece about 1 cm. wide, as shown at *S*.

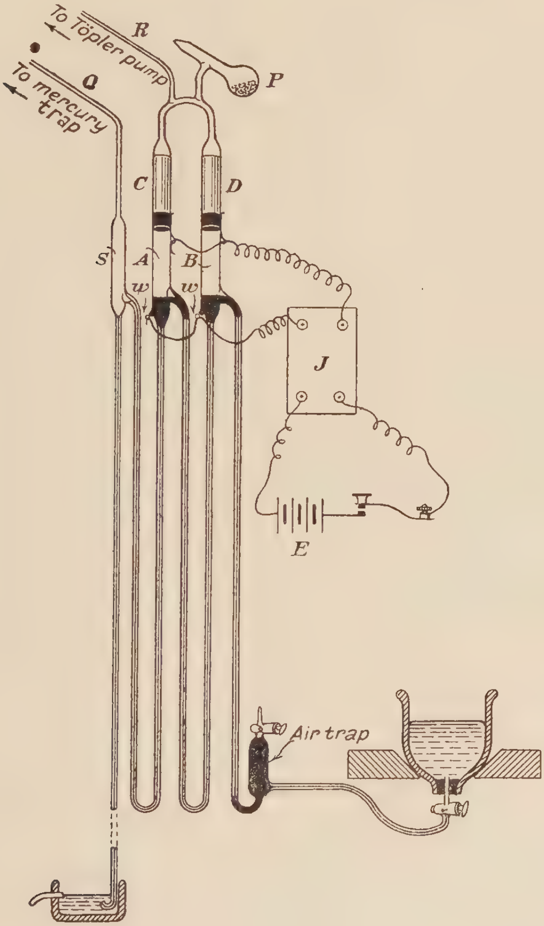


FIG. 1.

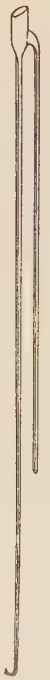


FIG. 2.

(*β*) When the vacuum is high, the pellets of mercury as they drop in the fall tube, are often shattered, and as a result the minute gas bubbles which are being removed, are enabled to re-enter the main apparatus. This somewhat serious imperfection can be almost entirely obviated by constructing the fall tube in two sections. The lower or main section is as usual cylindrical; its length is 72 cm. and its bore 0·8 mm. or less. The upper section must be made by the worker. It is formed by first thickening a suitable portion of a wide glass tube, and then drawing it out so that it is given a conical form. At the widest end, the bore should be about 3 mm., and at the narrowest 1 mm. The two tubes when fused together form a complete fall-tube as shown in Fig. 2.

It will be seen that the success of this device depends upon the fact that the pellets of mercury as they enter the conical part of the fall-tube, trap off in succession a maximum volume of gas; as the gas is pushed downwards, its column narrows continuously until the gas enters the cylindrical part of the tube; at the same time the pellets of mercury become fairly long and narrow cylinders, and these are not easily shattered. Ordinarily the minute quantity of gas trapped off towards the end, instead of forming a complete cylinder between the falling pellet and the column beneath, is seen to be present as an imperfect little bubble clinging to the wall of the fall-tube; if the bubble remains, it is ultimately joined by others which coalesce with it, and produce a complete disc-like bubble which then journeys on and is finally expelled.

DISCUSSION.

Prof. C. V. Boys said that the troubles which the author had dealt with had also occupied the attention of Rood some fifty years ago. The latter had sought to overcome them by arranging for the mercury first to fall in drops in a vacuum chamber to rid it of gas, and subsequently, at the end of the effective part of its fall, to impinge on the side of a tube so curved as to check the fall without shattering the drops.

Author's reply: Rood's device as described by Prof. C. V. Boys was ingenious, and would, according to my own experience, be distinctly beneficial. When drops of mercury fall within a vacuum, adherent gases are detached, and this alone leads to a decided gain in efficiency. But for my own particular work, the success of which demanded the absence of adsorbed gases and gas-grown skins, a more drastic scheme was imperative. The plans described in this present Paper, and also in earlier ones upon the same subject, appear to ensure a sufficiently complete removal of all matter capable of assuming the gaseous state.

XIII.—THE THOMSON EFFECT IN COPPER, IRON, AND CARBON STEELS.

By J. Young, *B.Sc., A.R.C.Sc., F.R.A.S.*, Lecturer in Physics in the University of Birmingham.

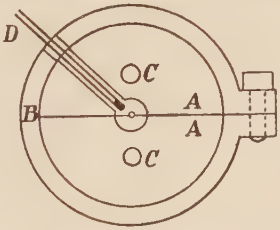
Received January 11, 1925.

Abstract.

The apparatus described by Nettleton in the Proceedings of the Physical Society, April 1922, has been employed for the study of the Thomson effect in copper, iron, and carbon steels. It was found that different results were obtained according as the electric current in the wire under test flowed with or against the temperature gradient. The results are compared with those obtained by other workers for the thermo-electric power of the materials investigated.

In view of certain magnetic and electrical phenomena which are exhibited by carbon steels at or in the neighbourhood of definite temperatures, for example, the change in magnetic permeability of such a steel at 215°C., it was suggested to me by Prof. S. W. J. Smith that similar changes very probably exist in other properties of the material and that it might be worth trying if any such variation was detectable in the Thomson effect. At that time (November, 1922) Nettleton had succeeded in perfecting his apparatus for the measurement of this effect in wires,* and it was considered opportune to use his method in this investigation.

Accordingly, an apparatus was constructed, consisting of two electric heaters, *A* and *B*, to the copper cores of which two copper discs, A^1 and B^1, could be tightly fixed by screws. Each disc was 4·8 cm. in diameter and 0·55 cm. in thickness. The experimental wire was silver-soldered to these discs, in the manner described by Nettleton, care being taken to see that the faces of A^1 and B^1 into which the wire penetrated were free from solder on their surfaces near the junctions. In this way the effect of Peltier heat creeping along the wire and so upsetting the temperature gradient in it was made as small as possible.



AA. *Copper disc split across diameter.*
B. *Steel band for clamping.*
CC. *Screw holes for fixing disc to heater.*
D. *Thermojunction.*

Fig. 1.

In some of the later experiments a method of fixing the wire by clamping was tried. The discs were split (*see* Fig. 1), the two halves being held together by a steel ring which could be tightened up by means of a screw. This method of holding the experimental wire was found most satisfactory, and it gave substantially the same results as were obtained in the experiments in which the wire was fixed by soldering.

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* Proc. Phys. Soc., April (1922).

The general method of experimenting has already been very fully discussed by Nettleton in his papers, but for completeness the following short description is given. A current, i_1, is passed through the experimental wire, the ends, A and B, being at steady temperatures, θ_1 and θ_2, respectively. When a steady state is reached, as indicated by a small bolometer coil wound centrally on the wire, the current is reversed and at the same time slightly reduced or increased to a value, i_2, such that, after an interval of about one minute, during which the apparatus is allowed to settle down, no change is detected in the temperature of the wire. This condition of balance is found by trial, the change, i_2-i_1, being adjusted by drawing plugs from a resistance box. The scheme of electrical connexions required to do this automatically by reversing the current is shown in Fig. 2. The principle underlying the method is thus to compensate the gain in Thomson heat on reversal by the loss of Joulean heat on reduction of the current from i_2 to i_1.

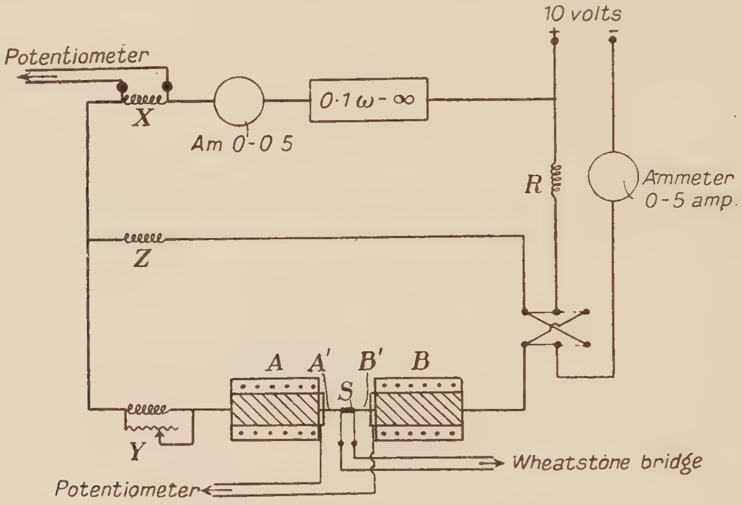

FIG. 2.—DIAGRAM OF APPARATUS.

The complete theory of the Thomson effect in an electrically heated wire was given by Nettleton in 1916.* He found that if the emissivity of the wire was sufficiently small,

$$\sigma_\theta = R(i_2-i_1)/JU \quad \ldots \ldots \ldots \ldots \quad (1)$$

where

σ_θ=coeff. of the Thomson effect at the temperature $\frac{\theta_1+\theta_2}{2}$.

R=actual resistance of the wire.

$U=\theta_2-\theta_1$.

J=mechanical equivalent of 1 calorie.

It is not proposed to discuss here the corrections which must be made when the emissivity ceases to be negligible.

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* Proc. Phys. Soc., 29, p. 68.

Under good working conditions, the emissivity must be small and nearly constant as the lagging between the two heaters will take up roughly the same temperature gradient as the wire itself. Nettleton apparently satisfied himself that under these conditions the use of the simple formula was justified, and it has been used entirely in the calculation of the results given in this Paper. In any case, some experiments were made in which the experimental wire had a different diameter, a factor which affects the emissivity term, and the results justified the use of the simple formula.

METHOD OF MAKING OBSERVATIONS.

The two heaters were first adjusted to approximately steady temperatures, differing by about 50°C., as indicated on two mercury thermometers inserted into holes in the respective heaters. The main current of 2·7 amperes obtained from a 10 volt battery was then started, and after a minute or two the bolometer coil was put into action. This coil had a resistance of about 2 ohms at ordinary temperatures and consisted of three layers of No. 40 silk-covered copper wire. Its length was usually 1 centimetre, and it was further insulated by being painted with Ohmaline varnish, a plastic varnish which was found most satisfactory even at a temperature of 300°C.

The Wheatstone bridge of which the bolometer coil formed the fourth arm was then balanced and the heating currents adjusted until the balance remained permanent. The galvanometer key was kept closed continuously during the latter observation. At first a slow creep of the galvanometer spot, due to the continual heating up of the asbestos packing protecting the experimental wire, gave great trouble. This, however, was got rid of to a great extent by shortening the experimental wire to about 4·5 cm., which, while lessening the sensitivity of the measurement, gives the more valuable advantage of steadiness and freedom from temperature storms. In some of the later experiments a compensating device was used to overcome this difficulty, a small coil similar to the bolometer and of equal resistance being placed in the asbestos packing and made an adjacent arm in the Wheatstone network. This arrangement gave great steadiness and was very convenient to work with. The current used in the bolometer was about 0·05 amp. and was supplied from a 2 volt accumulator. A Broca galvanometer of 20 ohms resistance was used to balance the bridge.

The next step was to make sure that the main current altered by a known amount when reversed. This was accomplished, as in Nettleton's experiments, by means of a special circuit which could be shunted across the main apparatus or a dummy apparatus, alternatively (*see* diagram), the change of current in the experimental wire being equal to the current in this shunt circuit. The circuit was adjusted at this stage by altering the resistance, Y, until the fall in potential across a standard ohm, X, remained the same on reversal of the commutator key, K.

The balancing of the Thomson and Joulean heats was then attempted. The bolometer being again put into action and the galvanometer key being kept depressed permanently, the effect of reversing the main current was tried. The deflection of the galvanometer spot after about 15 secs. gave some idea of the want of balance, and by adjusting the shunt current to a suitable value a balance of sufficient accuracy could generally be obtained for the purposes of these experiments. A difference of 1 in 100 in the shunt current usually produced a small perceptible effect.

The temperature difference between the ends of the experimental wire, i.e., between the two copper discs, was determined from the readings of two copper-constantan thermocouples of which the calibrations were known. The hot junctions of these couples were soldered with silver to the faces of the respective copper discs in contact with the heaters, the wires being brought out through horizontal slots and insulated with strips of mica. The thermo-electric E.M.F.s were read on a Clark-Fisher potentiometer to the nearest 5 micro-volts—this accuracy being amply sufficient for the purpose in view. The mercury thermometers were also read as a check, but, of course, gave readings considerably lower at the higher temperatures owing to stem errors and partly to slight temperature gradients in the heaters themselves.

To find the resistance of the wire under test, the fall in potential between the copper discs was measured when the main current of 2·7 amps. traversed the experimental wire, the shunt current being broken for this purpose. This has the advantage that no change need be made in the arrangement and that the resistance is found under the actual conditions of the experiment. Owing to the Seebeck E.M.F.s at the junctions of the wire with the copper discs, the fall in potential has to be read for both directions of the main current. It is easy to see that the mean of the two readings gives the fall in potential due to the resistance of the wire. The value of the main current was read off from a Weston millivoltmeter with suitable shunt, the accuracy obtainable being about one in a thousand. The value of the shunt current was obtained by noting the fall in potential across the standard ohm, X, the reading of a Weston milliammeter being used in setting the potentiometer.

Discussion of Results.

The Thomson effect was measured throughout the range 40-300°C. in Armco iron and in three carbon steels containing respectively 0·15, 0·60, and 1·1 per cent. of carbon. Before starting the experiments each of the specimens was annealed from about 700°C. Measurements were also made on a specimen of pure annealed electrolytic copper, for a purpose to be mentioned hereafter. Some preliminary measurements were also made on hard drawn Armco iron between the temperatures 40°C. and 180°C., but were not considered so reliable as those mentioned above.

Each of the specimens was used in the form of wire 0·122 cm. in diameter, and except in the case of copper the experimental length was 4·5 cm. The copper wire had an experimental length of 6·0 cm., partly to bring the shunt current within a reasonable value by increasing the resistance of the specimen, and partly as a greater sensitivity is practicable owing to the high heat conductivity and consequent freedom from temperature storms. The results with copper fully justify this alteration.

A fact of considerable interest and importance became apparent as the experiments progressed. This arose from the discovery that the reversal of the heaters (i.e., making the heater at the lower temperature in one set of experiments, that at the higher temperature in another set) gave a systematically different value of the Thomson effect. This was not due, as subsequent tests showed, either to any inaccuracy of the thermo-electric readings or to any asymmetry in the apparatus due, for example, to leakage. It was, in fact, due to a real asymmetry in the specimen itself induced probably by inhomogeneity. On reversal of the heaters in the case of the higher carbon steels, the difference was observed to be greater, a result considered confirmatory of the inhomogeneity theory. Indeed so great was this effect in the 1·1 per cent. carbon steel, amounting to 50 per cent. of the mean at 60°C., that it seemed very questionable if taking a mean of the two results would give a reliable value of the true Thomson effect. However, as the asymmetry decreased very considerably at higher temperatures being only 2 per cent. at 300°C., it seemed reasonable until the point was further investigated to take the mean of the results found on reversing the heaters. It is significant that the experiments with copper showed no sign whatever of this asymmetry.

The experimental results for iron and copper are given in Tables I and II, and some of the results are shown graphically in Figs. 3 and 4. Fig. 4 shows the mean, results, on the same scale, for each of the five specimens.

TABLE 1.—*Armco Iron.* (*Annealed.*)

i	P.D.	$i_1—i_2$	θ_A	θ_B	$\theta=\frac{\theta_A+\theta_B}{2}$	σ_θ
2·740 amp.	0·02337 volt	0·1026 amp.	184·4°C.	146·9°C.	165·7°C.	$—5{\cdot}58\times10^{-6}$
2·743	0·02127	0·1184	161·9	118·7	140·3	5·09
2·735	0·02543	0·1107	212·0	169·9	191·0	5·86
2·725	0·03129	0·0913	275·3	234·3	254·8	6·12
2·715	0·03308	0·0888	296·6	254·1	275·4	6·10
2·770	0·02695	0·1100	226·9	183·7	205·3	5·93
2·765	0·02864	0·1100	247·9	202·4	225·2	6·00
2·765	0·03618	0·0774	316·4	275·8	296·1	5·97
2·750	0·03894	0·0684	345·0	303·5	324·3	5·58
2·760	0·03726	0·0728	327·6	287·4	307·5	5·86
2·755	0·03480	0·1016	261·7	311·0	286·4	6·23
2·750	0·03030	0·1086	219·4	264·1	241·8	6·39
2·750	0·03314	0·1009	247·7	294·0	270·9	6·27
2·745	0·03062	0·1063	223·7	268·2	246·0	6·38
2·744	0·02768	0·1052	194·9	234·8	214·9	6·36
2·735	0·02695	0·1049	187·4	227·2	207·3	6·20
2·715	0·03675	0·0858	286·2	332·4	309·3	6·02
2·720	0·03751	0·0789	293·3	338·0	315·7	5·82
2·770	0·03836	0·0783	294·0	338·8	316·4	5·80
2·775	0·03664	0·0863	278·3	323·0	300·7	6·10
2·780	0·02609	0·0964	174·1	209·4	191·8	6·14
2·780	0·02879	0·1040	201·4	241·9	221·7	6·36
2·775	0·02103	0·1127	113·5	152·8	133·2	5·20
2·775	0·02350	0·1190	142·5	184·1	163·3	5·80
2·768	0·02230	0·1136	129·9	169·0	149·5	5·60
2·760	0·01759	0·1071	70·7	107·8	89·3	4·40
2·757	0·01759	0·1060	70·8	107·8	89·3	4·37
2·752	0·01948	0·1129	96·4	135·0	115·7	4·95
2·745	0·01425	0·0920	24·0	60·3	42·2	3·15
2·740	0·01518	0·1016	37·6	76·0	56·8	3·51
2·725	0·01436	0·0976	65·7	24·6	45·2	2·99
2·745	0·01837	0·1118	120·6	80·8	100·7	4·50
2.757	0·01858	0·1167	124·0	82·0	103·0	4·48
2·760	0·01644	0·1177	95·1	51·1	73·1	3·81
2·755	0·02035	0·1164	147·1	105·2	126·2	4·92
2·745	0·02260	0·1084	174·2	135·3	154·8	5·49
2·745	0·02520	0·1053	205·8	166·2	186·0	5·85
2·780	0·02708	0·1047	223·3	182·7	203·0	6·00
2·765	0·02871	0·1024	242·9	201·6	222·3	6·17
2·750	0·03182	0·0936	277·4	235·9	256·7	6·24
2·735	0·03647	0·0687	320·8	284·1	302·5	5·97
						calories/ coulomb-degC.

TABLE 2.—*Copper.* (*Annealed.*)

l	P.D.	$l_1 - l_2$	θ_A	θ_B	$\theta = \frac{\theta_A + \theta_B}{2}$	σ_θ
2·650 amp.	0·003035 volt	0·1086 amp.	27·0°C.	93·0°C.	60·0°C.	$+0·450 \times 10^{-6}$
2·685	0·003280	0·0993	55·6	115·6	85·6	0·485
2·675	0·003515	0·0927	79·5	139·9	109·7	0·482
2·685	0·003708	0·0992	97·4	161·6	129·5	0·510
2·690	0·003900	0·1049	116·7	181·7	149·2	0·560
2·680	0·004133	0·0973	140·5	203·4	172·0	0·568
2·675	0·004430	0·0940	173·0	235·7	204·4	0·594
2·670	0·004723	0·0924	202·4	266·4	234·4	0·612
2·670	0·005050	0·0894	236·5	300·0	268·3	0·636
2·718	0·005405	0·0912	260·7	325·0	292·9	0·675
2·750	0·003700	0·1076	152·6	84·8	118·7	0·511
2·745	0·004493	0·1017	232·4	165·2	198·8	0·590
2·740	0·005415	0·0998	325·4	256·0	290·7	0·680

The first point which should be noticed is that the Thomson effect increased more or less systematically with the rise in the percentage of carbon in the steels. This, of course, is to be expected from the thermodynamic relation

$$T\frac{d^2E}{dT^2} = \sigma_A - \sigma_B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where σ_A and σ_B are the Thomson coefficients for two metals A and B,

E = thermoelectric E.M.F. of a couple composed of these substances,

T = absolute temperature,

on account of the fact that $\frac{d^2E}{dT^2}$ increases when the percentage of carbon in the steel is increased (Belloc, Ann. de Chim. et de Phys., 1903).

The second point to which attention should be drawn is the sudden bend which takes place in the iron and steel curves about 200°C. In this connexion it is worthy of notice that various changes have been detected in other properties of iron near this temperature. For example, Thomson and Whitehead* have found a change in the electrical resistance of pure iron and in its thermoelectric power against platinum at 220°C. ; a heat evolution is reported by Monius and Scott† near 200°C. ; while Mayneord, working in Prof. Smith's laboratory, has shown that there is a falling-off in the magnetic permeability of iron in fields of about 0·07 gauss at a temperature of about 200°C. and ending about 300°C., these temperatures varying considerably with the heat treatment of the specimen. This latter observation is of special interest as the fall in permeability seems to coincide with the flat minima shown in the Thomson effect curves, while Prof. Lea has shown‡ that Armco iron has a maximum tensile strength at 230°C. The flatness of the minimum between 210°C. and 290°C. is very noticeable in the steel curves, but it would be rash to conclude that this is due to the presence of carbon. It has been suspected, however, that Cementite does not lose all its magnetic properties at 215°C., and that some gradual change may take place throughout this region.

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* Proc. Roy. Soc., Vol. 120, p. 587, Feb. (1923).

† Chem and Met. Eng., 1069 (1920).

‡ Engineering, 30, p. 829, June (1922).

COMPARISON WITH OTHER THERMOELECTRIC DATA.

According to the thermodynamic treatment, first applied to thermoelectric phenomena by Lord Kelvin, and which lead to his discovery of the effect which bears his name, equation (2) should stand the test of experimental verification. The data afforded by the present measurements on iron and copper and by the numerous researches which have been made on the thermoelectric powers of these

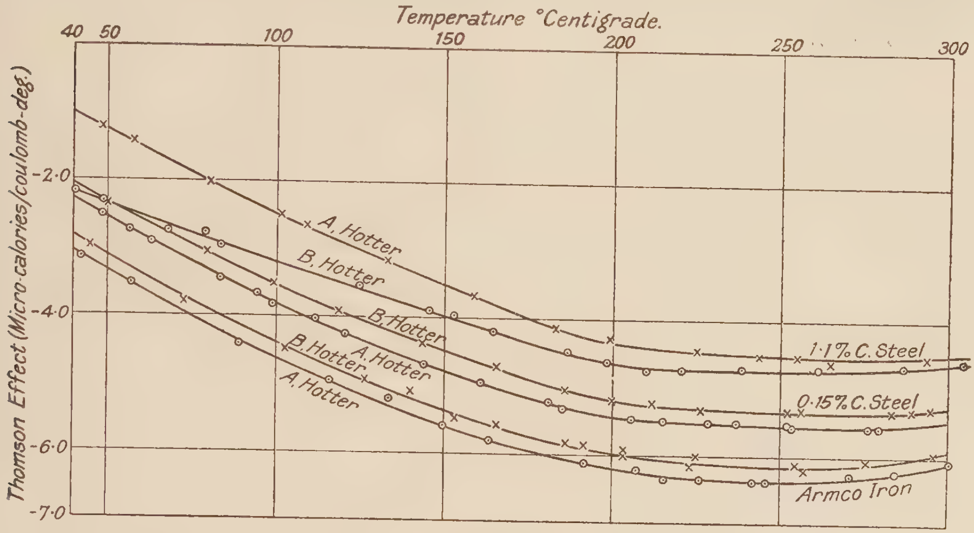

FIG. 3.

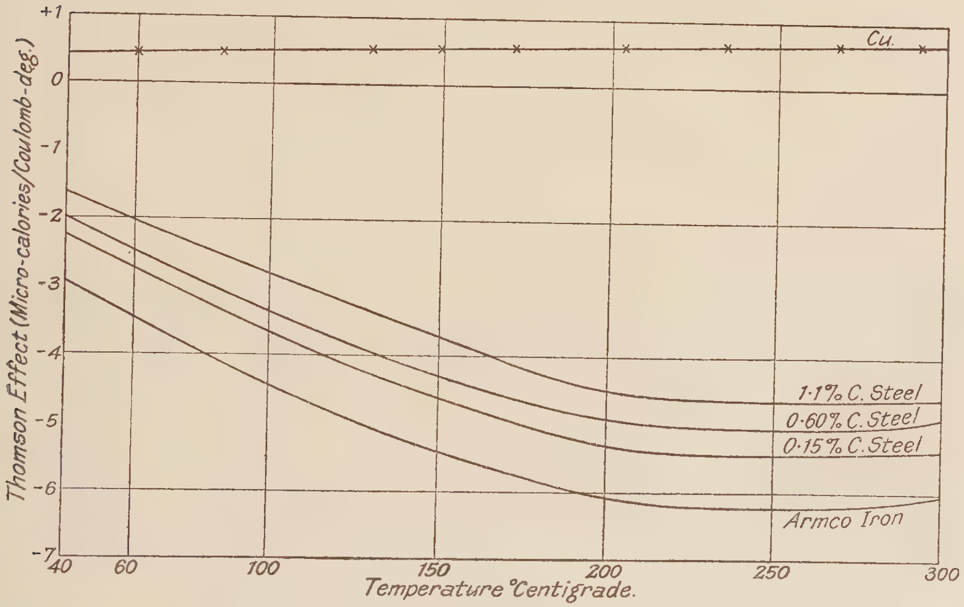

FIG. 4.

two metals at different temperatures provide a means of testing the equation. One of the most valuable researches is that of E. P. Harrison (Phil. Mag. III, p. 177, 1902), who has investigated the thermoelectric force of iron against copper over the range —200°C. to 1,000°C. Broniewski, in addition (Comptes Rendus, 156, p. 1983, 1913), has investigated the thermoelectric properties of the carbon steels.

In Fig. 5, the values of $\frac{\sigma_{Fe}-\sigma_{Cu}}{T}$, obtained from the present measurements, are plotted against the temperature, and Harrison's and Broniewski's results for $\frac{d^2E}{dT^2}$ are shown on the same diagram. Recognising the difficulty of measuring both the Thomson effect and $\frac{d^2E}{dT^2}$ accurately, in absolute units, it may be said that, on the whole, there is an approximate agreement between the Thomson effect measurements of the author and the thermoelectric power measurements of other workers. Nevertheless, further experimental confirmation of equation (2) was considered very desirable. As the apparatus for measuring the thermoelectric power is very easily set up, and as the samples of iron and copper were available from which were cut the specimens used in the Thomson effect measurements, some experiments were carried out on the thermoelectric power of these two metals. One of the copper-constantan thermocouples used in the previous research was taken to measure the temperature intervals, thus ensuring the same temperature scale as in the Thomson effect experiments.

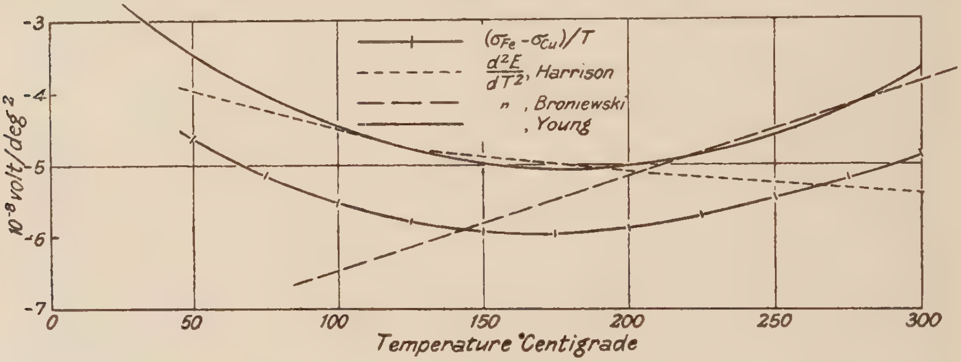

FIG. 5.

The values of the thermoelectric power in volts/deg. were plotted against the temperature, and the resulting curve showed more than a suggestion of a point of inflexion about 180°C. By trial, the following empirical equation was found to represent the results within the limits of experimental error :—

$$\frac{dE}{dt}=11{\cdot}15\times10^{-6}-2{\cdot}00\times10^{-8}t-1{\cdot}70\times10^{-10}t^2+0{\cdot}315\times10^{-12}t^3 \quad . \quad . \quad . \quad (3)$$

giving

$$\frac{d^2E}{dt^2}=-2{\cdot}00\times10^{-8}-3{\cdot}40\times10^{-10}t+0{\cdot}945\times10^{-12}t^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The latter equation gives a minimum value of $\frac{d^2E}{dT^2}$ at 180°C., the corresponding minimum in the curve derived from the Thomson effect measurements being at 178°C. This agreement is in itself striking, and is further confirmed by the parallelism of the two curves shown in Fig. 5, but there is, nevertheless, one serious disagreement between the two results. The Thomson effect measurements, while exhibiting the same general characteristics as those derived from the thermoelectric power, are numerically greater by an amount which is well outside the limits of experimental error. Whether this is to be ascribed to some omission in the theory of the measurements, or to an actual negation of the truth of the thermodynamic relation, is a matter for further consideration and experiment. It is significant, however, that constantan (the results for the Thomson effect in constantan were obtained by Dr. Nettleton) also shows the same peculiarity, and it is hoped to carry out shortly some experiments on nickel which may also confirm this observation. In addition, it may be noted that Latimer* has attempted to verify equation (2) from specific heat data, and while obtaining results which agree to 5 or 10 per cent. for most metals, finds considerable discrepancies for the ferromagnetic metals, that for iron being of the same order and sign as that in the author's investigations.

It seems possible that a Righi-Leduc effect may partly account for such a discrepancy, as in a soft iron wire the induction due to the current itself may be considerable, and would give an effect of the correct sign. It is difficult, however, to calculate the change in temperature of the bolometer coil which may be ascribed to this cause, and in consequence to determine the discrepancy, but an estimate gave a result only one quarter of that observed. The author's own opinion is that the similarity in the character of the Thomson effect and thermoelectric results points rather to the existence of some modifying cause such as that suggested above, than to an absolute denial of the irreversibility of the thermoelectric circuit. It seems quite possible that the Thomson coefficient may depend on the current, and, therefore, that the comparison of the thermoelectric power measurements which are made for zero current, with those of the Thomson effect which have been made for a current of 2·7 amp., is not justifiable.

In conclusion, I should like to express my indebtedness to Prof. Smith, not only for allowing me the privilege of working in his laboratory and placing apparatus at my disposal, but also for his kindly advice and encouragement during the progress of this work. I also wish to thank Dr. H. R. Nettleton, of Birkbeck College, London, for his courtesy in allowing me to see his apparatus under working conditions, and for the trouble which he took in explaining his methods.

Note on an Asymmetrical Heating Effect in Steel Wires.

[*Added* 27*th February*, 1925.]

In this note it is proposed to give a short account of the nature of the asymmetrical effect observed in the steel wires used in the preceding measurements of the Thomson coefficient and to put forward a probable explanation of the phenomenon.

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* Jour. Amer. Chem. Soc., 44, p. 2136 (1922).

In the first place, the phenomenon may be observed when the two ends of the wire are at the same temperature and when the temperature gradient in the wire is produced by the electrical current alone. If, under this condition, the electrical current is accurately reversed, an alteration in the temperature of the centrally wound bolometer coil is observed. Measurements of the potential difference between the ends of the wire were made for both directions of the current, and these were found to differ by less than one in a thousand. Yet to balance the asymmetry a current of 2·7 amp. had to be altered by 0·03 amp., which corresponds to an alteration of the heating effect of 2·2 per cent. Very careful tests were made to ensure accurate reversal of the current, and every precaution was taken to prevent leakage. The wire was also reversed longitudinally with respect to the rest of the apparatus, in order to make quite certain that the asymmetry was a property of the wire and not of the apparatus. The wire was also tested at various positions along its length, and although a considerable variation in the magnitude of the effect was observed, yet nowhere did any single wire show contrary effects in its different parts.

The results of these tests proved undoubtedly the reality of the phenomenon, and suggested that it could only be produced by an alteration in the radial temperature gradient on reversal of the current. Thus, for one direction of the current, heat is apparently produced near the surface and absorbed in the interior, while on reversal heat is absorbed at the surface and evolved in the interior. The ultimate reason why the effect is observed at all is because heat produced near the surface of the wire has a better chance of being conducted into the detecting coil and less chance of escaping along the wire than heat produced in the interior.

The variation of the phenomenon for different regions along the wire also suggested that its cause would probably be found connected with inhomogeneity of structure. This led to a photomicrographical examination of longitudinal sections of the wire, and it was soon discovered, as had been previously suspected, that there was a small but marked asymmetry in the structure as revealed by this means. An inspection of the accompanying photomicrograph, which shows the whole thickness of the wire (X 50), reveals the interesting result that, while the longitudinal markings near the top edge are on the whole inclined to the axis at a small angle, those near the bottom edge are inclined in the opposite direction. This gives to the section a "herring-bone" structure which, though not strikingly conspicuous, is nevertheless quite apparent when a careful examination is made. A number of measurements of the inclination gave an average value of about 2° near the edge. The structure is evidently related to the direction in which the wire is drawn.

Now, when a current passes through a wire there will be Peltier effects at the surfaces of the laminar constituents, of which the wire is composed, and if the thicknesses of the laminæ are unequal it is very easy to see that a "herring-bone" structure will cause the surface of the wire to be cooled, while the centre is heated, or vice versa. It remains to establish the fact that the Peltier effects required to produce the phenomenon are of a reasonable magnitude.

Let l = electrical current.

a = radius of the wire.

d = distance at which two adjacent laminæ of the same kind come to the surface.

Longitudinal Section of 0·71 per cent. Carbon Steel Wire (×50), showing Asymmetrical Structure.

To face page 154.]

Let pd=length of surface occupied by one of the thinner laminæ.

P=Peltier coefficient.

θ=angle of asymmetry.

Let heat evolved per unit length of surface of wire per unit time=H.

$$H=\frac{2p\sin\theta}{a}lP$$

A corresponding quantity of heat will be absorbed throughout the interior of the wire, and l^2R will be evolved due to Joulean heating, where R=resistance per unit length.

Let nH of the heat produced at the surface and mH of that produced throughout the interior reach the coil.

Then heat supplied to coil per sec. per unit length

$$=nH-mH+ml^2R.$$

Hence, for constancy of temperature in the coil, we must have

$$(n-m)\frac{2p\sin\theta}{a}Pl_1+ml_1{}^2R=-(n-m)\frac{2p\sin\theta}{a}Pl_2+ml_2{}^2R$$

From which we get

$$\frac{dE}{dT}=\frac{(l_2-l_1)mRa}{(n-m)2p\sin\theta\,.\,T}$$

Now, n will be greater than m.

Taking the values $n=2m$, $\theta=2°$, $l_2-l_1=0{\cdot}03$ amp.

$p=0{\cdot}33$, $T=313$, $R=0{\cdot}00204$ ohm/cm.

we get $$\frac{dE}{dT}=0{\cdot}50\times10^{-6}\text{ volt/deg.}$$

This corresponds to a difference of about 0·1 per cent. in the amount of carbon in the two constituents. This calculation shows that the Peltier effects produced in the wire are quite sufficient to cause an effect of the order observed.

There are two further pieces of evidence, however, which show that this theory is almost certainly the correct one. In the first place, some information can be obtained from the way in which the bolometer coil reacts to the reversal of the current when the asymmetry has been balanced. The galvanometer key being kept depressed, the reversal of the current causes an immediate deflection in the direction of the asymmetry, which is afterwards compensated by the Joulean heat. This shows that the effect which causes the alteration in the temperature of the wire must take place at the surface of the wire.

In the second place, it will be observed that the effect decreases more or less uniformly with temperature until at a temperature of 300°C. it is only one tenth of its value at 40°C. This is precisely the way in which the Peltier effect between two steels of different carbon content varies with temperature.

Incidentally, it rules out the possibility that the asymmetry is caused by Peltier effects at the points where the wire is fastened to the copper heaters because the

Peltier effect becomes zero at a temperature of about 260°C., and will consequently show reversal at 300°C. No such reversal of the asymmetry occurs.

Only a very few experiments were made to determine the relation between the asymmetry and the direction in which the wire was drawn. These experiments showed that if the wire was drawn through the die from right to left (thus producing a conical structure in which the apices of the cones pointed to the left) a heating effect was observed when the current passed in the same direction, and a cooling effect when it passed in the opposite direction.

In addition, it was found that the greatest effect took place in those portions of the wire which were last to pass through the die in the process of drawing. It will be easily seen that when such a wire is clamped between two copper discs the Peltier heat at the junction between the disc and that end of the wire in which asymmetry is greatest is of exactly the opposite sign to the asymmetrical heating effect. This was found to be the case when tested experimentally.

These experiments confirm the theory which is put forward in this note as to the nature of the asymmetry, and they clearly rule out the possibility of their origin arising from Peltier heat produced at the ends of the wire.

DISCUSSION.

Dr. D. **Owen** said that he would like to see very large numbers of experimental results of the kind obtained by the author, as those contained in the Paper were not numerous enough to support any one theoretical explanation exclusively. The results might, for instance, depend on the geometrical form of the specimens.

Dr. L. **Simons**: Throughout his work upon which that of the present author was based. Dr. Nettleton was fully alive to the difficulties involved in the heterogeneity of an ordinary drawn metallic wire. Would it not be possible to prepare a large single crystal of a pure metal, carefully cut in a definite direction with regard to a crystallographic axis, and then use the specimen for the determination of the Thomson coefficient ?

Prof. F. L. **Hopwood** said that if the asymmetry detected by the author was to be explained by the theory he had put forward (depending on the structure of the wire), it might be expected that the effect would be emphasised by torsion. Had he noticed any hysteresis effect ?

Prof. C. V. **Boys** suggested that the author should draw his own wires and note the relation between his asymmetrical results and the direction of drawing.

Mr. F. E. **Smith** suggested that the wire manufacturers would no doubt be willing to supply specimens labelled with information as to the direction of drawing.

Dr. H. R. **Nettleton** (communicated): I should like to congratulate Mr. James Young on the successful way in which he has carried out his measurements on the Thomson effect. Many points in his Paper interest me, and I should like to mention one or two. I note that in his later work Mr. Young has been using as his "isothermal equipotential ends" split discs which are clamped together. I am at present engaged in attempting to measure the Thomson effect in wires under tension, and I have long abandoned the use of solder and am using rectangular copper blocks, which are pressed together and force the wire into a cavity of small diameter. Like Mr. Young, I find this most satisfactory, my only difficulty arising when the wire thins under heavy strain. I am pleased also to observe that Mr. Young is using a compensating coil. Recent experience has taught me that the best compensating arrangement consists of a single turn wound on the experimental wire alongside the main detecting coil, all four leads being brought out side by side, embedded in wool, to their terminals. Mr. Young's method of measuring the resistance under the very current used for the reversals (instead of a current of 1·0183 amperes) saves a vast amount of time. It would seem, too, on the face of it, to be theoretically better, but I have experienced mathematical troubles in attempting to prove it rigorously. I should, nevertheless, use it myself if my potentiometer enabled me to read easily above 30 millivolts. I should like to know if Mr. Young's resistance measurements, when plotted against mean temperature, show any sign of bend in the neighbourhood of 220°C. I am interested in Mr. Young's remarks on heterogeneity which is doubtless the greatest obstacle he has encountered in his research. There is very little reliable information on the absolute value of the Thomson effect, and I hope that Mr. Young will be able to continue his valuable research work.

AUTHOR'S REPLY :—(To Dr. Owen.) The reason for examining the Thomson effect in steels is given in the introductory paragraph of my Paper. Although I recognise the importance of confining attention to pure metals, I would point out that the properties of mixtures are not undeserving of attention. The very interesting problem of the asymmetrical effect in the steel wires, for example, arises out of such experiments. I have not yet tried sufficient experiments for the purpose of finding how the asymmetry varies with the length and diameter of the specimen, to give any definite results on that point.

(To Prof. Hopwood.) A certain amount of temperature hysteresis has been detected in the experiments with iron, the Thomson effect being found to vary slightly with the heat treatment of the specimen. With the steels, however, this effect was very much less marked.

(To Prof. Boys.) I regret I am in the position of not having drawn my own wires. I undertook the research at the suggestion of Prof. S. W. J. Smith, who very kindly provided me with the wires from some of his own specimens.

(To Dr. Nettleton.) I have to thank Dr. Nettleton for his remarks. I have examined my resistance measurements, and find no trace of any sudden change in curvature in the "mean temperature-resistance" curve for iron. There are, of course, small and sudden changes in the resistance of iron at various temperatures, one of which is referred to in my Paper; but the effect of measuring the resistance of a wire under a temperature gradient and plotting this against the mean temperature would be to smooth out any irregularity of this kind.

XIV.—IMPROVED CATHODE-RAY TUBE METHOD FOR THE HARMONIC COMPARISON OF FREQUENCIES.

By D. W. Dye, *B.Sc.*

Received February 11, 1925.

ABSTRACT.

Convenient arrangements for the superposition of a telephonic and a radio-frequency displacement of the light spot of a Cathode-Ray Tube are described. The arrangements include the following :—

1. Circular or elliptical trace at a telephonic frequency to serve as a time axis. When great steadiness and uniformity are desired, a valve-maintained tuning fork controls the rate of revolution of the light spot.

2. Radial, circular and straight-line displacements at low radio or high audio-frequencies are next described and examples of the stationary patterns produced when these are superposed on the circular or elliptical time trace are given.

3. A special application of the method to the delineation of wave forms consists in the superposition of a long narrow elliptical time displacement and a straight-line high-frequency displacement in a direction at right angles to the major axis of the ellipse. By this means a considerable portion of the time ellipse is nearly a straight line representing a uniform time axis. The wave shape at the high frequency then appears in normal form and may be made of quite considerable size.

Some examples of each of the foregoing methods are given in the form of reproductions from prints made directly on pieces of bromide paper held down on the top of the Cathode-Ray Tube for a few seconds (3 to 10).

A special case occurs when an oscillatory valve system is arranged with grid and anode circuits closely coupled and harmonically resonant to each other. Wave forms consisting of a fundamental and a resonant single harmonic can be produced in this manner. Examples of current wave shapes in the grid and anode circuits are given, together with the voltage wave shape induced in a loosely coupled untuned coil.

THE cathode-ray tube has been used by a number of investigators for the delineation of wave forms and the determination of harmonic ratios of radio frequencies.

Various methods have been adopted to provide a convenient time base and to arrange so that confusion of the trace obtained is avoided.

Of the methods adopted, mention may first be made of the use of Lissajous* figures for the harmonic comparison of two radio frequencies. The chief limitation of this method is that ratios greater than about 15 : 1 introduce such complicated patterns that their interpretation and photography is difficult.

A method worked out in great detail by Dufour† consists in producing a zig-zag time trace across and down the screen. The high-frequency wave is superposed upon this in a direction at right angles to the time trace. For certain kinds of work this method is valuable, and allows results to be obtained which otherwise would be impossible. The equipment is very elaborate and expensive, and does not conveniently permit of a stationary pattern being formed. It is, therefore, necessary to take a photographic impression directly upon the plate inside the tube.

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* Lissajous Figures. F. Braun, Wied. Ann., 60, p. 552 (1897) ; and E.T.Z., 19, p. 204 (1898). Simon and Reich, Phys. Zeit., 2, p. 285 (1901). "Primary Radio Frequency Standardization by Cathode-Ray Oscillograph," Science Paper No. 489, Bureau of Standards (1924).

† A. Dufour, "Cathode-Ray Oscillograph : A Modified Braun Tube Arrangement," Jour. de Phys. et le Radium, 1, p. 147 (1920).

This latter necessity is, of course, inseparable from any method in which photography of a transient or non-recurring phenomena is required. In many cases, however, the oscillations under measurement or investigation are steadily maintained. It then becomes possible to produce a stationary pattern by arranging so that the path followed by the ray is repeated after a short time interval.

More recently Kipping* has used a neon tube trigger arrangement for producing a time trace which possesses some advantages and is very simple. In Kipping's Paper the idea of using the rotating ray method about to be described is suggested amongst others, but the idea originated a considerable time before with the present writer, and was successfully used at the National Physical Laboratory about 18 months ago. As long ago as 1899 Zenneck used a rotating ray for low-frequency measurement. E. Lübke† has also used a rotating ray for the purpose of delineating wave shapes at high frequencies ; in this method the exact analogy of the point by point method is used. The circularly rotating ray acts as a switching device by first passing through a radial slit, and thence between a pair of plates. The instantaneous conduction thus provided between the plates allows a voltmeter or electrometer to become charged to the corresponding instantaneous voltage. By slowly rotating, a few degrees at a time, the system producing the circularly rotating ray, the unknown wave-shape may be plotted out. The method of course, requires the special attachments within the tube to enable the system to work. The tube was of the heated cathode type, and operated with 400 volts anode potential.

The rotating ray method—which does not seem to have received much attention —can, however, be used with great advantages in many cases. In particular the method is valuable for the purpose of setting a low radio frequency to an exact value, for purposes of calibration, &c. The method consists simply in producing a circular or elliptical time trace by means of two-phase voltage of constant and low frequency.

The radio frequency may then be caused to operate on the ray in one of three different ways. These are as follows :—

1. A small voltage (about 20-30 volts) at the radio frequency is introduced into the steady anode voltage supply to the cathode-ray tube. By this means the ray receives a radial vibratory displacement due to the radio frequency, at whatever position it may be in its circular or elliptical path at the audio frequency. This is due, of course, to the variation in velocity of the electrons forming the ray, consequent upon the variation of anode voltage.

2. A small circular movement may be imparted to the ray at the radio frequency whilst the large circular or elliptical movement is provided by the audio frequency.

3. A vibratory movement, in a fixed direction, may be given by the radio frequency, whilst the large circular or elliptical movement takes place at the audio frequency.

The choice of these three methods is determined by the particular object in view in any given case.

Convenient circuit arrangements for carrying out these three methods have been worked out to suit the Western Electric Tube with heated cathode, and are described below after a short account of the best means of producing the rotating ray.

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* M. V. Kipping, " A Practical Demonstration of Some Applications of the Cathode-Ray Oscillograph," Wireless World, p. 705, Mar. 5 (1924).

† E. Lübke, " Measurements of Wave Form with Braun (Hot Cathode) Tube," Archiv. für Elektrot., 6, p. 161 (1917). Translation by writer, Electrician, 83, p. 270 (1919).

The most accurate method of producing the circular time base is to use a valve-maintained tuning fork as a fundamental time standard. A small voltage obtained from the fork is amplified up by a two-stage amplifier, so as to provide a source of voltage of very constant frequency. This voltage is converted to a two-phase source by the help of a condenser and a virtual non-inductive resistance. These two voltages of adjustable amplitude and phase relationship are applied to the two deflecting plate systems of the cathode-ray tube.

A convenient set up for the method is shown in Fig. 1 in diagrammatic form.

The tuning fork is shown at A, and is of ordinary form. The small voltage given by the output winding will not in general be of good wave form. In order to improve this the anode circuit of the second valve of the amplifier is a tuned circuit, consisting of the air core self-inductance L_1 and adjustable condenser C_1. The coil L_1 is best made with a generous amount of wire, so as to be of fairly good time constant, with inductance of about 0·5 henry. A secondary winding closely coupled to L_1 provides the supply of nearly sine wave form. This is connected to the phase-splitting arrangement whereby the two quadrature components are produced.

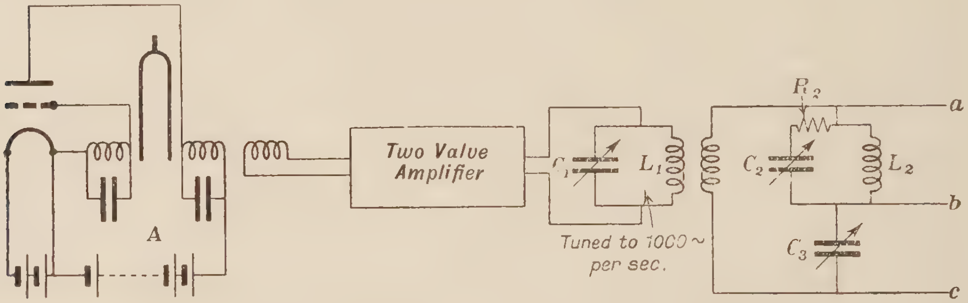
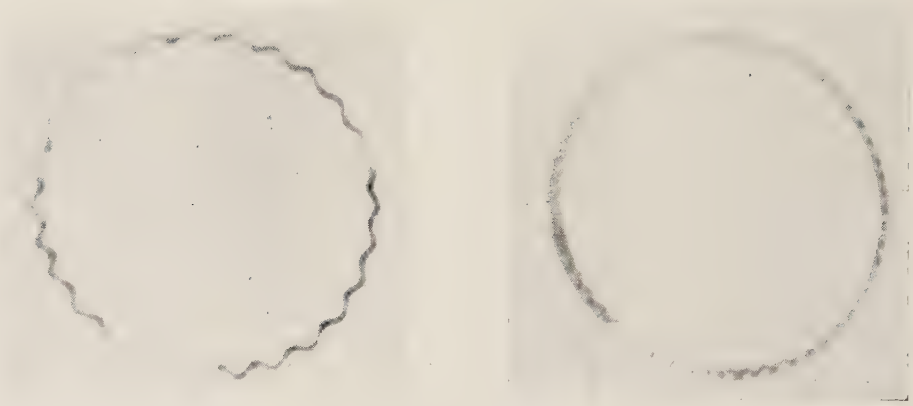


FIG. 1.—ARRANGEMENT FOR PRODUCING CIRCULAR TIME TRACE.

The phase splitting arrangement consists of the condenser C_3 in series with a tunable resonant circuit L_2, C_2. This circuit when adjusted to resonance behaves as a pure non-inductive resistance of value adjusted by the resistance R_2. It has been found desirable to use a virtual resistance of this form in preference to an actual resistance. The reason for this is that when an actual resistance of say, 10,000 ohms is used, this resistance has also a high impedance at the radio frequency. It then becomes almost impossible to keep a small parasitic radio frequency voltage from finding its way on to the deflecting plates connected to this resistance. By the device indicated a truly non-inductive resistance is provided at the audio frequency, whilst at any other frequency not very near the resonant one the impedance is very low.

The two quadrature voltages are applied to the deflecting plates, and by adjustment of C_2 and R_2, a nearly perfect circle is obtained after tuning C_1 to resonance.

The circle is adjusted to any desired size by adjustment of the brightness of the filament of the second valve of the amplifier. It can, of course, be made an ellipse



(*a*) Radial high frequency deflection of 26,000 cycles per second on a circular time base at 1,000 cycles per second.

(*b*) Radial high frequency deflection at 26,500 cycles per second on a circular time base at 1,000 cycles per second.

Fig. 4.

[*To face page* 161.

of any eccentricity by slight detuning of L_2, C_2, or by adjustment of R_2 or C_3. In cases where it is not important to know the radio frequency exactly, the tuning fork may, of course, be dispensed with, and a low-frequency oscillator used instead. The freedom for the choice of a suitable rotation-frequency is then a great advantage in many cases.

The valve oscillator and coupling coils are as shown in Fig. 2. This, at first sight, appears to be of normal type, but the arrangement of the capacities and windings L_1, L_2 permits of the production of resonant harmonic wave forms as discussed later on. The coils *A*, *B* are for the purpose of applying the radio frequency voltage to the cathode-ray tube in a number of different ways to suit different requirements.

When required, coil *B* is connected to a pair of coils wound on a double ebonite bobbin of such form and proportions that they produce a uniform magnetic field which is transverse to the axis of the tube. These coils are shown in diagrammatic form in Fig. 3, at F_1, F_2, which indicates the general cathode tube arrangement. The bobbin carrying these coils is made a sliding fit on the parallel part of the tube, and can be rotated so that the direction of the magnetic field can be altered in azimuth.

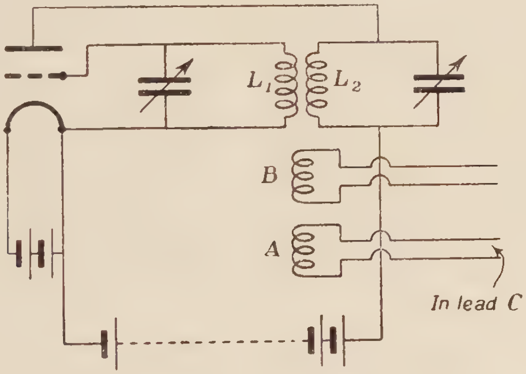


Fig. 2.—Valve Oscillator with Resonant Harmonics.

Coil *A* provides a radio frequency voltage which may be introduced either into the anode battery circuit of the cathode ray tube at "*d*" in Fig. 3, or in the circuit of one pair of deflecting plates at "*e*."

An example of the pattern produced by the superposition on the time circle of a frequency of 26,000 cycles per second introduced as a voltage in the anode circuit is shown in Fig. 4(*a*). This figure is reproduced from a photograph made by holding a piece of bromide paper down on the top of the cathode tube for about three seconds. The adjustment of frequency must, of course, be very precise in order that only a negligible drift occurs during three seconds. If, for example, a drift of one-tenth of a cycle is permissible during the three seconds, the constancy and steadiness of the high-frequency source must be equal to about two parts in a million. This can be secured by allowing everything time to settle down, and then by catching the picture just as it passes through a stationary condition corresponding to harmonic synchronism.

The value of frequency is given directly by counting round the number of waves in the circle and multiplying by 1,000. The method not only allows exact frequencies, such as 26,000 to be obtained, but a frequency such as 26,500 or 26,333 can also be set with perfect certainty. There is considerable difficulty in photographing these fractionally harmonic synchronous frequencies, since the light spot has to make two or three revolutions before retracing its path. A correspondingly longer exposure is necessary, and at the same time the permissible drift is reduced. A reproduction of such a case is shown in Fig. 4(*b*). There is, however, no difficulty in setting a frequency to such values; in this manner a considerable number of standard reference frequencies is available for calibrating purposes.

One of the disadvantages of this method of superposition is that a rather high voltage at the radio frequency is necessary to produce the requisite radial deflection; a loss of definition occurs when the voltage reaches its minimum value. The radial movement permissible is also very limited.

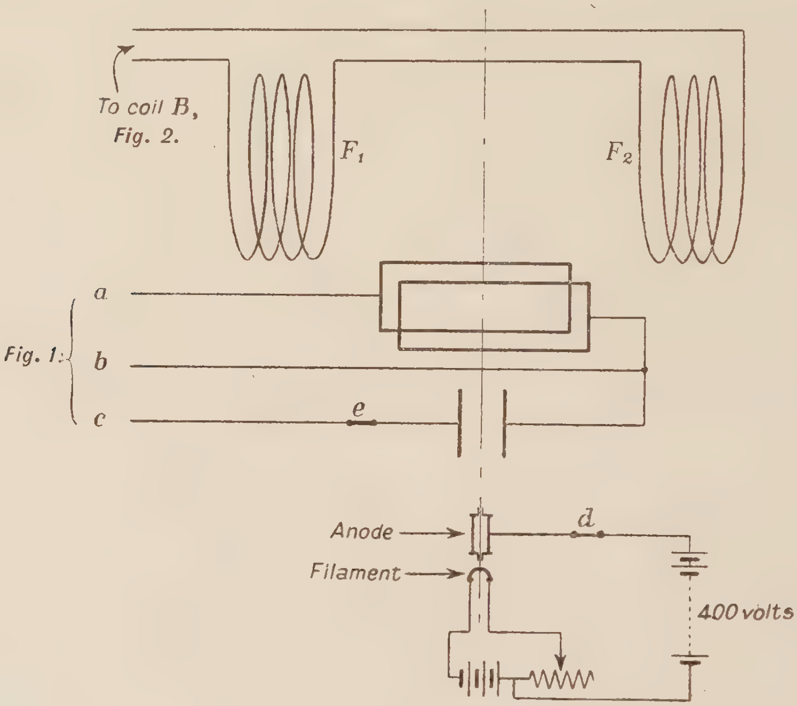


Fig. 3.—Cathode Ray Tube Arrangement.

Superposition of Circular Motions.

The arrangement for this method can be seen from Fig. 2 and Fig. 3. The high frequency rotation of the light spot is produced by the help of the field coils F_1, F_2, previously described. These produce a magnetic deflection in a plane which can be rotated by turning the double bobbin about the axis of the tube. The quadrature deflection is produced by introducing the high-frequency voltage given by

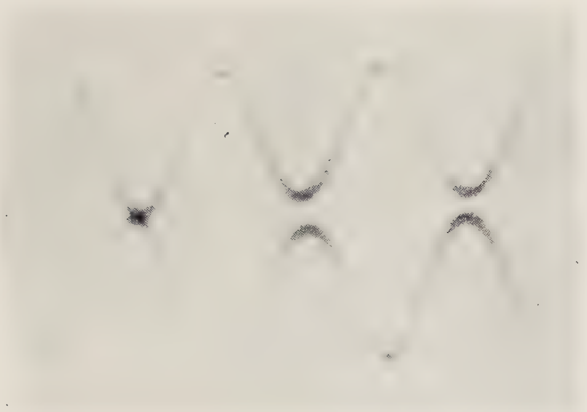
(*a*) Circular high frequency at 26,000 cycles per second rotating in same direction at the circular time base of 1,000 cycles per second.

(*b*) The same as (*a*), but the two circular motions are in opposite directions.

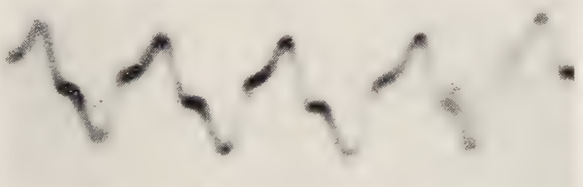
(*c*) Conditions as at (*a*), but the amplitude of the high-frequency motion has been adjusted to give a cusp.

FIG. 5.

[*To face page* 163.



(*a*) Sine wave at a frequency of about 15,000 cycles per second. Long elliptical time base.



(*b*) Same frequency as (*a*), but with a considerable proportion of a third harmonic.

FIG. 6.

[*To face page* 163

inducing coil *A* into the circuit at the point "*e*" of Fig. 3. This voltage can also be obtained from the ends of the deflecting coils F_1, F_2, by suitably choosing the inductance of these.

The phase adjustment is made by turning F_1, F_2, about the axis of the tube. The amplitude adjustment is made by sliding the coils F_1, F_2 up and down on the tube or by variation of coupling of the inducing coils *A* and *B*. The small high-frequency circle is obtained and adjusted with the large low-frequency circle cut off.

When both circular motions are allowed to operate simultaneously the beautiful circular looped patterns shown in Fig. 5 (*a*) and (*b*) are obtained. Two cases arise, the two circular motions may be in the same or in opposite directions. The corresponding patterns are shown in Fig. 5 (*a*) and (*b*), reproduced from photographic prints taken in the same manner as Fig. 4.

An inspection of the two pictures given in Fig. 5 shows that for the same high frequency the number of loops is two greater in one case than in the other. When the two circular motions are in the same direction, the loops are on the inside and the number of them is one less than the ratio of the frequencies. When the two circular motions are in opposite directions, the loops are on the outside and the number of them is one more than the ratio of the frequencies. By adjustment of the relative diameters of the two circular motions it is possible to obtain a sharp loop or cusp resulting from the instantaneous arrestment of the spot. This condition is very satisfactory photographically, and also renders the setting of fractional harmonics more certain. One of these latter is shown in Fig. 5(*c*) for a case corresponding to (*a*). Although it is well nigh impossible to photograph them, it is not difficult to set frequencies corresponding to seven or eight journeys of the light spot before the cycle is repeated. For example, a frequency such as 24,875 can be set by this means. The advantage of this method over the foregoing method is the much smaller high-frequency voltage necessary and the formation of loops rendering counting or photography easier.

The arrangement is, however, not convenient for the examination of wave forms. For this purpose, therefore, the following method (3) is recommended. This method is merely a particular case of (2) just described. One of the components of the high frequency is suppressed—by removal of the deflecting coils F_1, F_2, for example. The *h.f.* circular motion becomes a straight line of fixed direction. The *l.f.* circular motion is then made elliptical with the major axis perpendicular to the *h f.* direction. By making the eccentricity considerable we can greatly open out the time scale, and also render it nearly a straight line for a considerable length. The time scale will, of course, not be uniform right round the ellipse, but if it is long and narrow there will be a portion in the middle of the nearly straight part of sufficient length and uniformity of velocity to be quite satisfactory for delineating wave shapes at moderately low radio frequencies without distortion. The ordinate motion at the radio frequency may be made large when this method is used, and a reasonable size of wave obtained.

Examples of a good wave and of a wave with harmonics are given in Fig. 6. The nature of the waves with harmonics and the special uses of such waves will follow later.

The applications of the foregoing methods to telephonic frequencies are obvious ; it is merely necessary to use a slower time base corresponding to a frequency of revolution of, say, 100 ∼ per second, instead of 1,000 ∼. For quantitative work it is very desirable that the two motions should be proportional to their respective quantities; they should also be rectangular. For these reasons the high-frequency voltage should be introduced into the lead marked *C* on Fig. 3 at the point "*e*," and the elongation of the ellipse must then be obtained by reducing C_3 to about a quarter of its original value. The rectangularity of the high-frequency motion with regard to the major axis of the ellipse can be seen by eye when the two motions are taking place together, but at an inharmonic ratio. Adjustment is made by condenser C_2.

The limits of frequency which may be standardized are determined largely by the tube itself. Owing to the relatively low velocity of the ray, the time occupied by the electrons during passage between the deflecting plates is such that loss of definition occurs at frequencies above 200,000 per second.

This value cannot, however, be directly compared against a fundamental time base of $\frac{1}{1000}$ of a second owing to uncertainty of counting. This trouble can be entirely overcome by the use of resonant harmonics. A brief account of these is given in Section II. of the Paper.

SECTION II.

RESONANT HARMONICS IN VALVE OSCILLATORS.

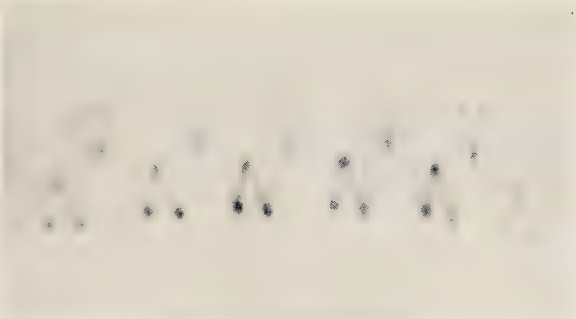
The distorted wave shown in Fig. 6 was known to be of a kind which commonly occurs in small valve oscillators such as heterodyne wavemeters and sources.

It is, of course, well known that curvature of the valve characteristics and changes, with amplitude, of the internal anode and grid impedances can, and indeed must, introduce harmonics into the wave shape produced, but in the case shown and in many other cases the distortion was so large that it was felt that other special causes were responsible for the distortion.

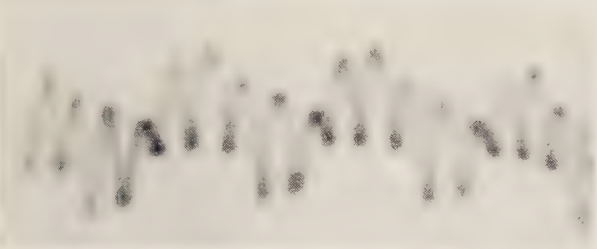
The matter was therefore followed up somewhat. Observations of a power-valve oscillator at the laboratory showed that when working in a normal manner the wave form was very good. The generator was such that normally the oscillatory circuit was of very small decrement and was in the anode circuit of the valve. The grid winding was of thin wire and had such mutual inductance to the oscillatory coil that the voltage on it was about one-fifth that on the oscillatory coil. The coupling between the coils was close. There was, in this case, no condenser across the grid winding. When, however, the main oscillatory circuit was in the grid circuit of the valve with the fine-fire coil as inductance whilst the heavy-wire coil was in the anode circuit, the distortion was very pronounced.

A small variable condenser was accordingly placed across the anode winding. The value of this condenser was slowly increased whilst at the same time the frequency was kept constant by a careful simultaneous adjustment of the grid oscillatory circuit condenser. A remarkable series of changes occurred in the wave shape thus held approximately steady on the oscillograph. Harmonics such as the ninth, eighth, seventh, sixth, fifth, &c., successively became reinforced and reached large values as the anode condenser was varied.

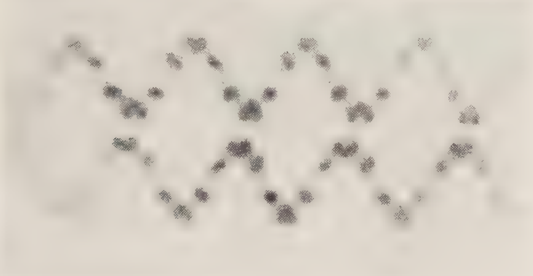
When a harmonic is at its maximum amplitude the wave shape appears to consist almost entirely of the fundamental and this harmonic; such cases are shown in Fig. 7, (*a*), (*b*), (*c*) and (*d*). At intermediate settings of the anode condenser the



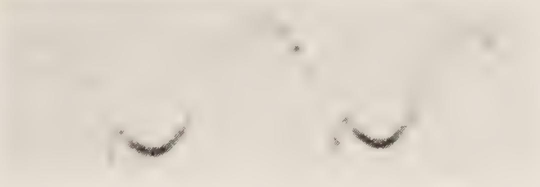
(*a*) Fundamental of 24,000 cycles per second with strong third harmonic.



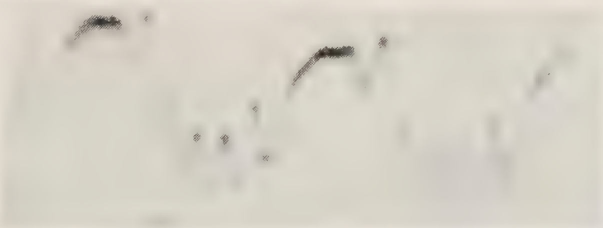
(*b*) Similar to (*a*), but with a resonant fifth harmonic.



(*c*) Fundamental of 25,000 cycles per second with resonant harmonic giving 175,000 cycles per second.



(*d*) Showing interference between two neighbouring odd and even harmonics.



(*e*) A case similar to (*d*) for the seventh and eighth harmonics of a fundamental of about 12,000 cycles per second.

FIG. 7.

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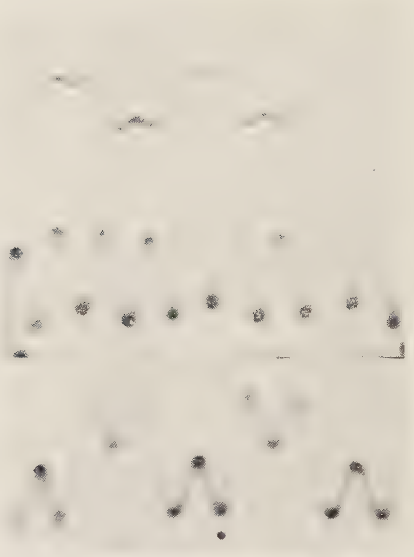

(*a*) Current in grid coil circuit of oscillator. Fundamental of about 20,000 cycles per second.

(*b*) Current in anode coil under exactly the same conditions as (*a*).

(*c*) Voltage induced in a coil loosely coupled to the oscillator.

Fig. 8.

[*To face page* **165**.

wave shape passes through curious forms. For example, Fig. 7 (*d*) shows a shape corresponding to about equal proportions of two adjacent harmonics and very little else. These two harmonics reinforce each other during one-half of the wave and cancel each other out during the other half. In some cases the harmonic oscillation does not build up and die down smoothly during each half-wave, but commences with a jerk as shown by the curious peak marked " *a*." A similar case for the seventh and eighth is shown in Fig. 7 (*e*).

These harmonic resonances correspond undoubtedly to a setting of the condenser of the anode oscillatory circuit such that the resonant frequency of this circuit bears an integral ratio to the main grid oscillatory circuit frequency. It is to be expected that the valve is able to produce the two corresponding frequencies simultaneously. It might be thought that two inharmonic frequencies could also be produced in the same manner, but such does not appear to be the case. The transition from one wave shape to another is smooth and continuous. It is not possible to obtain a stationary pattern and get a blurred figure at the same time, such as would occur if the two frequencies were inharmonic.

The wave shapes shown in Fig. 7 represent electromotive force induced in a coil coupled to the oscillator by mutual inductance. The voltage induced in such a case is, of course, proportional to " ω," so that the harmonic components in the wave of induced voltage will be magnified in proportion to the order of the harmonic by comparison with the corresponding components of the current wave in the oscillator. The current wave in a practical case will therefore in general be much smoother than the waves of voltage shown in Fig. 7. In many cases, however, we are directly concerned with the voltage induced in a circuit. In such cases the wave shapes shown are those which must be considered.

In the actual oscillator used in these experiments we have two closely coupled circuits both carrying currents of different wave shapes and both coupled to the external coil in which the wave shape of voltage is produced. The voltage wave will therefore depend upon the two current wave shapes and the values of the two mutual inductances to the external coil. The phase relationship of the two current waves will also largely affect the induced voltage wave shape.

In Fig. 8 are shown three wave shapes corresponding to one condition of the oscillator and its circuits.

The curves " *a* " and " *b* " are current wave shapes in the grid and anode coils respectively. Curve *c* is the voltage wave induced in a coil. The current waves were obtained by inserting the deflecting coils F_1 and F_2 directly into the circuit concerned. The coils were then oriented round until the deflection was at right angles to the major axis of the ellipse.

Unfortunately it was not possible to observe the phase relationship of the two currents. This could, however, have been done by the aid of two pairs of coils with axes at right angles to one another. The time circle would not then be used. A small Lissajous figure would be obtained in which a distorted ellipse would be formed by the two fundamental components each with its harmonic.

Referring to Fig. 8 again, the current waves produced are those which might have been expected. The anode circuit, of harmonic resonant frequency and small damping coefficient, has a current wave shape of which the main component is the harmonic, but in which there is a noticeable amount of fundamental present. The grid circuit of resonant frequency of the fundamental has a greater damping coefficient than the anode circuit. As a consequence the wave shape has a considerable proportion of the harmonic component present.

The voltage wave shape is similar to those shown in the previous examples given.

The writer is not aware that this particular property of a valve oscillator has been observed before. For certain purposes such an oscillation is of great value. For example, a cascade system of oscillators all in harmonic synchronism can be built up whereby a very high radio-frequency may be controlled from a lower one.

For standardization purposes the method is particularly suitable. One can, for example, use a fundamental frequency of, say, 30,000 cycles per second with a resonant ninth harmonic and so obtain a frequency of 270,000 cycles per second in a manner which leaves absolutely no uncertainty regarding frequency.

DISCUSSION.

Mr. NORMAN KIPPING: A variety of the harmonic generator referred to in the Paper has for some years been in use in "Carrier" telephony ("Carrier Current Telephony and Telegraphy," E. H. Colpitts and D. A. Blackwell, Journ. A.I.E.E., Vol. 40, Nos. 5 and 6, May-June, 1921). In this case the most convenient arrangement has been found to be the use of an amplifying vacuum tube rather than an oscillating vacuum tube, for the introduction of harmonics. An oscillating vacuum tube would be used to generate, say, 5,000 cycles, the output being fed into an amplifying element in such a way as to overload it. For obtaining a supply of the various harmonics produced by any tube when overloaded (in this case 10,000, 15,000, 20,000 cycles, &c.) selective circuits would be employed. The original 5,000 cycle wave would also be amplified without distortion and transmitted along the transmission line. At the far end it would be introduced again into a distorting amplifier (by filters). Further selective circuits at the distant end would ensure that the carrier waves at the two ends of the transmission line were truly equal in frequency. Mr. Dye is to be congratulated very heartily on his Paper. His method of frequency calibration employing a rotating magnetic deflection is particularly clever. From the standpoint of wave-form examination Mr. Dye's method is less simple than my neon-lamp method mentioned in his Paper. The two systems have, however, about the same range. As Mr. Dye has referred in his Paper to my prior publication of the "cogged wheel" frequency calibration device, it may be stated that I was working on this quite independently. No doubt Mr. Dye was actually before me.

Mr. R. A. WATSON WATT: I would join in congratulating the author on his extremely elegant method for the comparison of frequencies of high ratio and over wide ranges, particularly when the application of the looped circular pattern is supplemented by the "double graduation" provided by the resonant harmonic method. Mr. Dye will perhaps not disagree with me in thinking the applications to wave-form delineation less important and less elegant. In work on the wave-form of atmospherics, Prof. Appleton, Mr. J. F. Herd and I passed from the use of an elliptic base of zero minor axis to the elliptic base of moderate eccentricity recommended by Mr. Dye, but we were glad to abandon both in favour of a device which we called the "Linotime," and which I regard as offering, for similar limits of dimensions of pattern, intensity of illumination, &c., a much closer and more conveniently attained approximation to linearity than do the open ellipse of Mr. Dye's scheme or the exponential curve of Mr. Kipping's neon-tube method. The device, to whose development my contribution is negligible in comparison to those of my two collaborators, and on which patent applications have been filed, comprises a retroactive triode oscillator with a condenser in the grid circuit. The leak across the grid condenser takes the form of a saturated diode, and in the absence of other leak conductances the curve of recovery from the well-known phenomenon of negative charging of the grid, interrupting oscillation, is strictly linear, and is applied as a time base. The base thus obtained is both linear and unidirectional, since the recovery period can be made so long relative to the oscillating-charging period that the latter, providing the back stroke of the time base, fails to excite notable fluorescence over a wide range of base frequencies. The control of base period can, of course, be made as accurate as in other methods. The theoretical advantages of a system in which the inherently linear law is departed from only on account of slight defects in the apparatus, as against devices dependent on regarding inherently elliptic or exponential laws as sensibly linear over small ranges, are borne out in practice.

It is not clear from the Paper how Mr. Dye avoids the vitiating effects of synchronisation phenomena, which will in general, on account of the coupling within the oscillograph, produce a stationary pattern even when the independent frequencies under comparison depart from an integral ratio by more than the few parts in a million mentioned by the author. It would be of value if Mr. Dye would explain how he ensures that his measurement is made on the undisturbed frequency to be determined.

Mr. T. SMITH demurred to the author's terminology in the description of Fig. 5 of the Paper. He held that the number of loops is always equal to the number of the partial under investigation provided that the loops be properly counted, the complete passage round the circle being reckoned as one loop with positive or negative sign.

Dr. D. OWEN, in congratulating the author, said that the new utility he had given to this oscillograph opened up attractive vistas of experiment and greatly increased the value of the instrument. Was there any reason for limiting the fundamental frequency to 1,000, or would it be possible to start from, say, 25,000 ?

Prof. E. V. APPLETON (communicated) : In connexion with the experiments on atmospheric wave-forms (Proc. Roy. Soc., A., 103, 84, 1923), it was very soon found that circular and elliptical time-bases were not nearly so useful as a linear and unidirectional base obtainable from a special type of triode oscillator circuit. By means of this oscillator (British patent application No. 3472/24), the cathode-ray spot can be made to travel from left to right uniformly in a controllable time, and then spring back to the origin almost instantaneously when the process is repeated. Vertical movements of the cathode spot are thus read off without distortion and without ambiguity as to time-sense. The electrical circuit used is an interesting example of a non-linear oscillator and the theory of its working, which depends on oscillation-hysteresis, has been fully worked out. An account of this will be published shortly.

This kind of time-base was developed in March and April, 1923, and has been in general use since then in all the stations carrying out wave-form observations for the Radio Research Board. It has also proved useful in experiments on the wave-form of the current pulses produced by the entry of α-particles into an ionization chamber (Proc. Camb. Phil. Soc., Vol. 22, p. 434, 1924). For the examination of periodic current or voltage changes the base is just as useful, since synchronization effects are usually sufficiently developed to keep the pattern stationary.

The " locking " of two oscillators, of which the frequency of one is n times the other, where n is integral, is well known, and has been used by Mercier for standardization purposes. The conditions under which this is possible are given in a Paper " On the Synchronization of Triode Oscillators " (Proc. Camb. Phil. Soc., Vol. 21, p. 231, 1922), where the formula (8) developed for the " locking " of fundamental periods applies equally well to harmonic synchronization.

AUTHOR'S reply (communicated) : The method of producing a resonant harmonic which I have described can, I think, hardly be classed with that referred to by Mr. Kipping. The method referred to by Mr. Kipping is, of course, well known, and has been used in principle for calibrating wavemeters for a long time, consisting, as it does, of the principle of first distorting a wave and then selecting the desired harmonic introduced by the distortion. In the method I have described, however, the two frequencies are simultaneously produced by one valve with only two oscillatory circuits and no other coils.

The method of delineating wave forms described in the Paper is certainly less simple than that using a neon tube, involving as it does the complication of an independent source and amplifying arrangements. Having, however, definitely required these parts for harmonic frequency determinations, it was considered of sufficient interest to show this further application of the apparatus.

I thank Mr. Watson Watt for his appreciative remarks and for his kindly criticism of the application of the method to the delineation of wave form. I am afraid, however, that, for the purpose in view, I must still maintain that the elliptical time base is much more satisfactory than any other form is likely to be. The particular point at issue in my use of the method is to determine the order of the resonant harmonic with regard to the main wave, and to be able to adjust so that any desired resonant harmonic within the range available is at its optimum value. It is necessary at the same time to adjust the fundamental high frequency so that it is an exact integral of the fundamental time base. I think it will be realised that for these conditions the method described is particularly suitable and satisfactory. But in cases of non-periodic phenomena such as atmospheric disturbances, and in many other cases, the method devised by Dr. Appleton is undoubtedly greatly superior.

With regard to the question of interlocking of the two frequencies, this, in the present case, is not a vitiating effect, but a desirable one. As a matter of fact, the effect is so small as to be negligible. The two frequencies will not interlock, even when one is different from a harmonic of the other by only one part in a million. The tuning fork has very great stability of frequency, and owing to the tuned circuit in the second stage of the amplifier, the wave forms of voltage applied to the cathode tube to produce the time circle are very nearly sinusoidal.

The radio-frequency source used in my experiments was operated on an anode potential of about 300 volts. It therefore possessed considerable stability of its own. This doubtless further contributed to the entire freedom from observable interlocking of the two systems.

Some of the points raised by Prof. Appleton have been dealt with in the reply to Mr. Watson Watts' remarks. The writer has read with great interest the Papers to which Prof. Appleton refers, and will look forward to the further Paper promised on the patented method of obtaining a uniform time trace, more especially in view of the fact that no hint of the method is given in either of the two Papers mentioned.

The author is, of course, quite familiar with the beautiful measurements made by Mercier on the absolute determination of the velocity of propagation of waves along parallel wires, in which a cascade system of oscillators all in harmonic synchronism is used. The present method, which only applies to frequencies within the limits of the oscillograph, does not make any use of the principle of harmonic interlocking of two frequencies, since the single-valve oscillator used cannot generate other than frequencies which are in harmonic ratio to one another.

Mr. T. Smith has called attention to a want of precision in regard to my explanation of the difference of two between the numbers of loops on the circles of Fig. 5 (*a* and *b*). I regret that in the reading of the Paper I did not express it in the same clear manner that Mr. Smith has done. Since, however, I have confined myself in the text to the mere statement of the results, no confusion of the reason can arise.

With regard to Dr. Owen's query as to the choice of a fundamental frequency ; there is, of course, no reason why any other frequency within the scope of the instrument should not be used. This is briefly referred to at the end of Section 1 of the Paper in regard to the use of a slower time base for telephonic work. The upper limitations are determined mainly by the limitations of the tube. For standardization purposes the choice of a time base of $\frac{1}{1000}$ sec. has been determined from considerations of accuracy and constancy. A suitably designed valve maintained tuning fork of a frequency of 1,000 periods per second is specially satisfactory from this point of view. Doubtless by the use of a piezo-electric quartz oscillator a sufficiently accurate time base of $\frac{1}{25\,000}$ second could be realised, but for the purposes of frequency standardization within the limits of the cathode-ray tube of the type used there would be no special advantage in this.

XV.—THE VARIATION OF YOUNG'S MODULUS AT HIGH TEMPERATURES.

By Jas. P. Andrews, *B.Sc.* (East London College).

Received December 19, 1924.

Abstract.

The variation of Young's Modulus with temperature is found for zinc, silver, phosphor-bronze, lead, and soda glass by a static method, to within about 150° of the melting-point. For these substances (as for several others recorded) the modulus varies exponentially with temperature, so that q(Young's Modulus)$=q_1e^{-b_1t}$ (where q_1 and b_1 are constants) up to a temperature roughly half-way from absolute zero to the melting-point, and $q=q_2e^{-b_2t}$ for the remainder. The values of the constants are tabulated.

Introduction.

THIS Paper continues the work on Young's Modulus previously published,* in which the variation of the modulus of elasticity of nickel, platinum, aluminium, and silica was determined up to temperatures not greater than 900°C.

Method.

A rod or wire of the substance, about 5 cm. long, was suspended vertically from a clamp at its upper end, within an electric furnace. The whole could be rotated any angle in a vertical plane, and a knowledge of this angle, together with the observation of the deflection of the free end of the beam or wire (either under its own weight or under load) enabled Young's Modulus to be calculated at any temperature of the furnace.

Apparatus.

The small electric furnace, which was previously closed with a copper end-piece, was during these experiments closed with a slab of uralite, to ensure a more uniform distribution of temperature. The slab was grooved to allow the passage of light through the apparatus, and the ends of the groove closed with glass windows.

A circular scale at the back of the furnace exhibited the angle of rotation, and upon it were stops to ensure exactly the same rotation on each occasion.

Fig. 1 shows the apparatus with the furnace rotated through 45°.

Temperature measurements were made with a small platinum resistance-thermometer, as shown in the diagram.

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* Lees, Andrews, and Shave, Proc. Phys. Soc., 36, p. 405 (1924).

THEORY.

Since the theory was given in the Paper already mentioned, it is only necessary to repeat the formulæ used on this occasion. These are :—

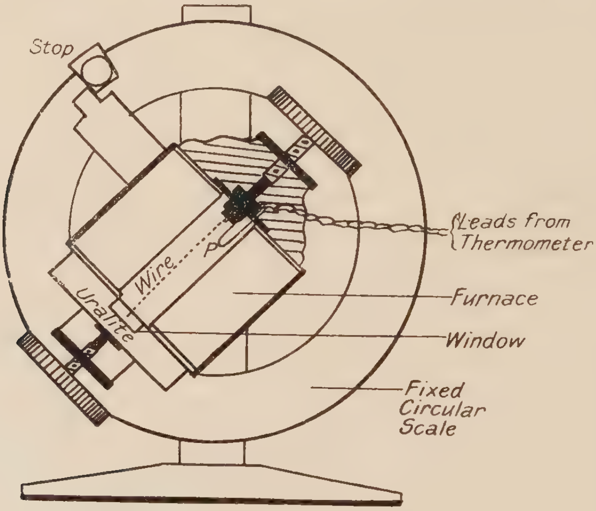
(1) For wires bending under their own weight,

$$(a)\ q=\frac{8gl^3m\cos\theta}{y_1d^4\pi}\quad\text{or}\quad(b)\ q=\frac{2\rho gl^4}{y_1d^2}\cdot\cos\theta$$

(2) When a weight is used whose centre of gravity is p from the free end of the wire,

$$q=\frac{64gl^2}{y_1d^4\pi}\left\{M\left(\frac{l}{3}+\frac{p}{2}\right)+m\left(\frac{l}{8}+\frac{p}{6}\right)\right\}^*.$$

where q=Young's Modulus, ρ=density, g=the acceleration of gravity, l=length of the wire, d=its diameter, y_1=the deflection of the end measured perpendicularly to the length of the wire, m=mass of the wire, M=the mass attached to the end, and θ=the inclination of the wire to the horizontal.



P= Platinum Thermometer.

FIG. 1.—DIAGRAM OF THE APPARATUS SHOWING ELEVATION WITH A SECTION OF THE FURNACE. THE OBSERVING MICROSCOPE WAS RIGIDLY CONNECTED WITH THE FURNACE, AND ROTATED WITH IT.

In several of the wires used, a measurable, progressive increase of length was noticed, taking place chiefly after viscous flow had set in. From time to time this increase was estimated, and a correction to the value of Young's Modulus obtained on the assumption that the volume remained constant, applied as follows :—

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* Owing to a misprint, the second term in the brackets, of this expression, appeared in the last Paper as $m\left(l+\frac{p}{6}\right)$.

From equation 1 (*a*) above

$$q=\frac{kl^3}{d^4} \text{ where } k=\frac{8\,gm\,.\cos\theta}{y_1\pi} \text{ a constant.}$$

But since the volume of the wire remains approximately constant,

$$d^2=\frac{\text{constant}}{l}$$

Hence $q=Kl^5$, where K is constant, and $dq=5Kl^4dl$,

$$\text{or } dq=\frac{5q}{l}dl$$

VISCOUS FLOW.

About halfway between absolute zero and the melting-point, the metals begin to exhibit viscous flow. Under these circumstances the deflection observed increases with the lapse of time. The calculation of Young's Modulus is therefore subject to some uncertainty unless the time taken in reading the deflections is very small. In the experiments the rotation of the furnace was rapidly accomplished, and the reading taken at the instant the rotation was complete, the end of the wire being kept in view the whole time. In spite of this, the last one or two readings must be regarded as doubtful in most cases. The probable correction required to Young's Modulus would be positive, thus raising the curve somewhat at highest temperatures.

RESULTS.

Zinc Wire (2).

Length, 4·847 cm., p. 0·1375 cm. Diameter, 0·0283. Used with weight M, 1375 gm.

$$\theta=70^\circ. \quad q=\frac{7{\cdot}654\times10^{12}}{y}$$

Temperature.	y	$q\times10^{11}$	Remarks.
18°C.	8·95	8·85	
62	10·4	7·35	Very slight viscous flow.
98	11·0	6·96	
128	12·0	6·38	
150	13·5	5·68	
170	14·5	5·28	
198	16·7	4·58	
225	18·1	4·23	
260	20·1	3·82	Viscous flow very rapid.
288	18·9	4·05	
318	21·9	3·49	

This wire was subjected to a second heating, but on removal from the furnace it exhibited a tendency to collapse, bending sharply at one or two points along the wire. It appeared to be very soft.

Zinc wires exhibit a rapid descent of Young's Modulus which begins at room temperatures. From the general resemblance between this variation, and that at higher temperatures in other metals, it may be inferred that the rapid decrease in the case of zinc begins at some point below room temperatures. It may also be observed that the rate of decrease falls off slightly at the highest temperatures. This is in accord with observation on several other metals.

Silver Wire (3).

Length, 5·600 cm. Diameter, 79·78 × 0·000353 cm.

$\theta=20°$. $\rho=10{\cdot}5$. $q=\frac{1{\cdot}057\times10^{12}}{y}$

Temperature.	y	q	Remarks.
30°C.	1·45	$7{\cdot}28\times10^{11}$	
127	1·50	7·03	
195	1·52	6·95	
260	1·62	6·52	
324	1·65	6·39	
388	1·70	6·21	
448	1·90	5·56	
500	2·02	5·22	
560	2·50	4·22	Slight viscous flow.
619	3·10	3·40	
680	3·75	2·81	Viscous flow considerable.

This wire and one other which was tried (which had previously been heated to about 650°C.) showed small excrescences when removed from the furnace.

Young's Modulus for silver decreases slowly with increase of temperature up to temperatues roughly midway between absolute zero and the melting-point (995°C.), after which the decrease is much more rapid. A tendency is again shown, however, for this decrease to be somewhat retarded at the highest temperatures.

Lead Wire (3).

Typical of all lead wires tried. Length, 5·60 cm. Diameter, 114·84 × 0·000353 cm.

$\theta=69{\cdot}5$. $\rho=11{\cdot}37$. $q=\frac{6{\cdot}091\times10^{11}}{y}$

Temperature.	y	q	Remarks.
20°C.	2·72	$2{\cdot}24\times10^{11}$	Distinct viscous flow.
46	3·02	2·02	
70	3·70	1·65	
96	4·15	1·47	
120	4·60	1·33	
146	6·75	0·905	Viscous flow very rapid.
172	10·0	0·609	

The variation of Young's Modulus for lead appears to be almost linear over the range taken; but it must be remembered that, however rapidly the observations were taken, for the last two or three values, the viscous flow was so considerable that some error will certainly have been made. (See above, Viscous Flow.)

The apparent linearity was observed by Koch and Dieterle* also (who employed an oscillation method), but the above results differ widely from theirs.† In the range 0°C.—150°C. Young's Modulus for the rolled strips they used, decreased by 17·1 per cent., whereas the decrease in the above experiments amounts to more than 60 per cent. Part of this difference may be due to the very different previous treatment.*

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* Ann. d. Physik., 68, 5, No. 13, p. 441, Aug. 31 (1922).

† It may be pointed out here that Koch and Dieterle calculated their values by means of the formula $N=\frac{m^2}{4\sqrt{U3}}\frac{a}{l^2}\sqrt{\frac{E}{S}}$ (quoted from Kohlrausch, Lehrb. d. Prakt. Phy., Abschnitt, 54a), which is obviously applicable only to undamped vibrations. The effect of friction will be to diminish N, the frequency, and the values given by Koch and Dieterle seem to need considerable correction. The tendency would be to diminish the variation they record for the elasticity.

It may be noticed that in this case, the elasticity after heating appeared to be slightly less than before. This need not be actually the case, as it is more than probable that a slight stretching of the wire took place while it was in the very soft condition.

Phosphor Bronze Wire (1).

Original length, 5·675 cm. Diameter = 0·02427 cm.
$\theta = 42°$. $\rho = 8·8$.
$q = \frac{2·701}{y} \times 10^{12}$. Correction for stretching, $dg = 5\frac{q}{l} \cdot dl$.

Temperature.	y	$q \times 10^{11}$	dl.	Correction.	$q \times 10^{11}$ corrected.	Remarks.
16°C.	2·70	9·48	...	...	9·48	
84	2·72	9·42	...	...	9·42	
152	2·90	8·86	...	...	8·86	
208	2·80	9·16	...	...	9·16	
273	2·85	9·00	...	...	9·00	
332	3·17	8·08	0·003	0·019	8·10	Viscous flow slight.
393	4·02	6·37	0·015	0·089	6·46	
464	5·40	4·74	0·026	0·114	4·85	Viscous flow rapid.
532	7·25	3·53	0·049	0·161	3·60	

Phosphor-bronze evidently follows the usual law as found for elementary metals—viz., the elasticity decreases slowly at first, but after reaching a temperature roughly half-way from absolute zero to the melting point the increase becomes more rapid, the rapidity decreasing again at the highest temperatures. The modulus of elasticity was found to be greater after heating.

The phosphor-bronze wires were only very slightly oxidised, only a slight colouring being visible after removal from the furnace.

Glass Fibre.

Drawn from a rod of Soda Glass.

Length, 5·575 cm. Diameter, 0·0166 cm. Mass m, 0·00308 gm.

Temperature.	y	q	Remarks.
25°C.	2·60	6·07	
88	2·65	5·96	
130	2·72	5·81	
211	2·82	5·60	
294	3·00	5·27	
337	3·05	5·18	
394	3·22	4·90	
454	3·40	4·63	Viscous flow slight.
517	4·00	3·94	Viscous flow considerable.

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* Koch and Dieterle used strips rolled out from wires.

The following measurements will give an accurate idea of the regularity of the fibre, which was only a chosen piece pulled out in the usual way from a rod. The measurements are of the diameter in scale divisions of the microscope, and are taken at four points dividing the wire into three equal parts :—

46·5, 47·0, 47·8, 47·0.

Glass was chosen as an amorphous substance, in contrast to the crystalline metals. It will be seen that the rapid decrease begins considerably nearer the melting point than in the case of the metals.

The above results are only typical ones, chosen from among many others, and the type of variation these numbers exhibit is shown in the following series of curves.

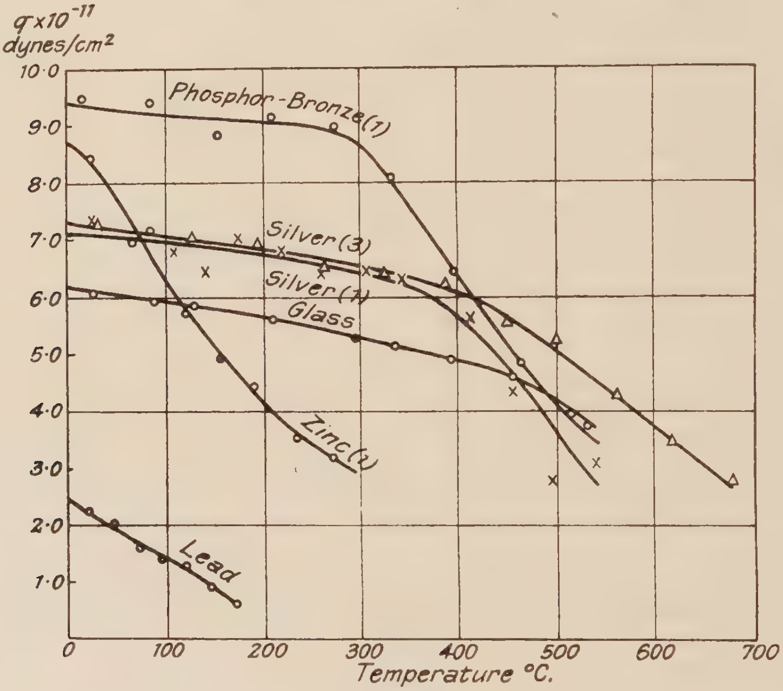

FIG. 2.

SUMMARY AND CONCLUSIONS.

With the exception of lead and zinc, the variation of Young's Modulus with temperature appears to be of the same type in all the substances tested. This variation consists of a slow decrease during the lower part of the temperature range, followed by a more rapid decrease at higher temperatures. It is probable that the apparently exceptional cases of lead and zinc follow the above rule also, but that the temperatures at which the variation is slow are below those used in these experiments.

It is found that if log q is plotted against temperature, practically all the curves consist of two straight lines. This remark applies equally well to the results of other investigators, some of which are included in the curves shown in Fig. 3. Accordingly, each curve may be represented by two equations of the type $q = q_0 e^{-bt}$. The following table gives the values of the constants q_0 and b as deduced from the present experiments and those of other investigators.

If $q = \left(q_1 e^{-b_1 t}\right)_{t_1}^{t_2} + \left(q_2 e^{-b_2 t}\right)_{4}^{t_3}$, where t is Centigrade temperature.

Substance.	Observer	$q_1 \times 10^{-11}$	b_1	Range °C.	$q_2 \times 10^{-11}$	b_2	Range °C.
Aluminium ...	J.P.A.	5·7	0·00131	0–100	8·5	0·00248	200–500
,, ...	K.&D.*	6·05	0·000555	0–200	10·0	0·00262	270–500
Brass	K.&.D.	7·35	0·000511	0–400	9·8	0·00119	440–600
Duralumin ...	K.&.D.	6·0	0·000348	0–300	7·7	0·000998	360–600
Glass	J.P.A.	6·2	0·000589	0–400	...	...	...
Gold	K.&.D.	5·5	0·000163	0–330	7·45	0·00103	400–600
Iron	K.&.D.	14·3	0·000392	0–470	...	...	...
Lead	J.P.A.	...	...	...	2·5	0·00612	0–150
,, ...	K.&D.	...	...	...	2·05	0·00108	0–250
Phosphor-bronze	J.P.A.	8·70	0·000137	0–240	26·2	0·00393	300–500
Platinum ...	J.P.A.	13·4	0·0000937	0–550	...	...	...
Silver	J.P.A.	7·4	0·000437	0–380	20·3	0·00288	450–680
Steel	...	...	...	...	...	...	...
Tin	K.&D.	...	...	...	3·6	0·00153	0–210
Tungsten ...	Dodge†	35·2	0·0000751	0–900	...	...	...
Zinc	J.P.A.	...	...	...	8·6	0·00300	0–300

It may be observed that the region in which the variation changes from the first to the second value occurs roughly at the same fraction of the absolute temperature of fusion. The list below indicates this. In the list T_1 represents the estimated temperature which is midway between the end of the slow variation and the beginning of the rapid. Brass and duralumin seem to be rather exceptional.

Substance.	Absolute temp. of fusion, T_0.	T_1 (abs.).	$\frac{T_1}{T_0}$
Aluminium	930	454	0·49
Brass	1,170 (approx.)	700	0·60
Duralumin	920	580	0·63
Gold	1,330	645	0·49
Iron	1,800	760	0·42
Phosphor-bronze	1,273	573	0·45
Platinum	2,030	900	0·44
Silver	1,235	670	0·54
Silica	2,070 (approx.)	1,050	0·51
Zinc	690	275	0·46

In the cases of lead and tin no such bend was found at ordinary temperatures. If we multiply the absolute temperatures of their melting points by the mean value of the numbers in the last column (omitting brass and duralumin) we obtain for the temperature of rapid decrease 12°C. and −36°C. respectively, which sufficiently accounts for their non-appearance. Dodge investigated tungsten up to 1,000°C., but found no similar bend in the curve. From the above general rule we should expect

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* Koch and Dieterle.

† H. L. Dodge, Phys. Rev., 11, 311, Ap. (1918). The numbers are deduced from a very poor curve given by Dodge.

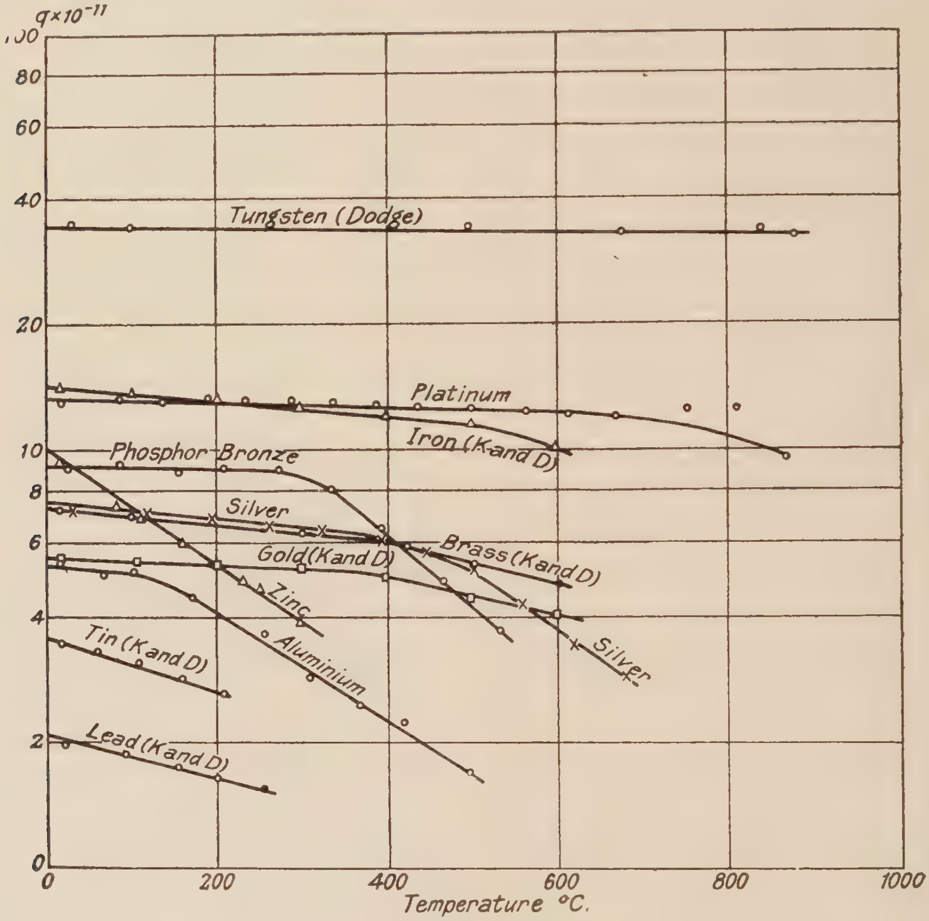

FIG. 3.

the decrease to occur at about 1,420°C., which again accounts for the apparent exception.

In conclusion, I have to thank Dr. C. H. Lees for advice during the course of this work.

DISCUSSION.

Dr. HARRISON said that the formulæ employed assumed a linear relation between stress and strain, whereas measurements carried out by himself some years ago showed that no such relation holds at temperatures above the change point. The wire would then become plastic and its form would cease to be an "elastica."

Dr. D. OWEN inquired whether the author had experimented with specimens of various lengths, as defects near the clamping point (where they would have a maximum effect on the results) would be checked by so doing. The value of Young's Modulus as usually found from the deflection of a loaded bar depends on whether the bar is clamped at one or at both ends, presumably for some such reason as the above. Had the author studied the relation between his results and the change with temperature of the crystal structure of the specimens ?

Mr. F. E. SMITH pointed out that the diameter of the phosphor-bronze wire had been given in centimetres to five significant figures, and questioned the validity of the last one or two of the latter.

Capt. C. W. HUME (communicated subsequently) : May not the discrepancy in measurements on bars referred to by Dr. Owen be due to neglecting the effect of shear strain as distinct from bending strain ?

The AUTHOR, in reply to Dr. Harrison, admitted that the elastic law does not hold at high temperatures and that the term " Young's Modulus " then ceases to be appropriate. The observations were taken immediately after the application of a given stress in order to obtain definite conditions as regards the effect of plasticity, and the results so obtained represent the ratio of stress to strain under the conditions described. They may be of value to engineers who have to deal with conditions which are similar, although they are not easy to interpret theoretically. In reply to Dr. Owen : Each result given in the Paper is a fair sample of a number of observations in which various specimens were used, though the dimensions of the apparatus did not permit of varying their length within very wide limits. It had not been possible to study the crystal structure of the specimens, but single crystals had been found to have an elasticity less than the average. The numerical result quoted by Mr. Smith had been copied into the Paper in error from a manuscript calculation. The fourth significant figure could be regarded as fairly reliable.

XVI.—CRITICAL VELOCITY OF FLOW PAST OBJECTS OF AEROFOIL SECTION.

By E. G. Richardson, *B.A.*, *M.Sc.*, *Ph.D.* (University College, London).

Received February 3, 1925.

Abstract.

By observations of the " Æolian Tones " of vibrators of aerofoil section, critical values for flow past an object of aerofoil section have been obtained of a fluid incident at various angles. The minimum value of VL/ν for unsteady flow is found to fall from 60 at 0° incidence to 45 at 20°, and then more rapidly.

Introduction.

THIS work is a continuation of that described in a previous Paper, entitled " Æolian Tones."* Some of the methods described therein have been applied to an examination of the " Æolian " vibration of brass and rubber models of aerofoils with a view to finding the lowest stream-volocity of fluid past the model at which a vibrator of suitable frequency, n, could be made to vibrate. It is explained in that Paper how an object of width D will vibrate in a plane at right-angles to the direction of the stream, if its natural frequency coincides with that of the eddies produced behind it, as given by the equation V/nD=constant, provided that at this velocity VD/ν exceeds a certain value, below which no eddying is produced in the wake of the object, when, in fact, the movement of the fluid is " steady." If the dimensions and tension of an aeroplane strut of such section should be such that one of its natural frequencies conforms to that of the vortices produced, it might conceivably fracture itself by the resonant oscillation set up ; cases of fracture of air-screw blades have been attributed to this cause. In the following observations the value of V for maximum resonance was determined as before when VD/ν was well above the limit ; but when the dimensions of the system were such that V lay near the critical value, the lowest speed for vibration to continue was observed, as the speed was slowly reduced, in order to get nearer to the critical velocity. Thus, in such a case, the lower extremity of the resonance curve was " observed" instead of the peak.

Brass Aerofoils in Liquid.

Corresponding values of V, n and D were found by the revolving tank method. The first aerofoil made had a diameter of 0·094 cm., and chord of 0·7 cm. It was about 20 cm. long, and was filed down from a piece of strip brass to the section shown in Figure. When this was clamped at one end, however, in the fashion used for the round rods, it was found to be too stiff to vibrate appreciably. Accordingly, a piece about 5 cm. long was sawn off and soldered at one end to a spring steel strip—an old hack-saw blade—20 cm. long. When about 3 cm. of the aerofoil dipped under water and the other end of the blade was clamped, the aerofoil was found to respond well to the vertical motion behind it, when the stream grazed the flat surface of the aerofoil.

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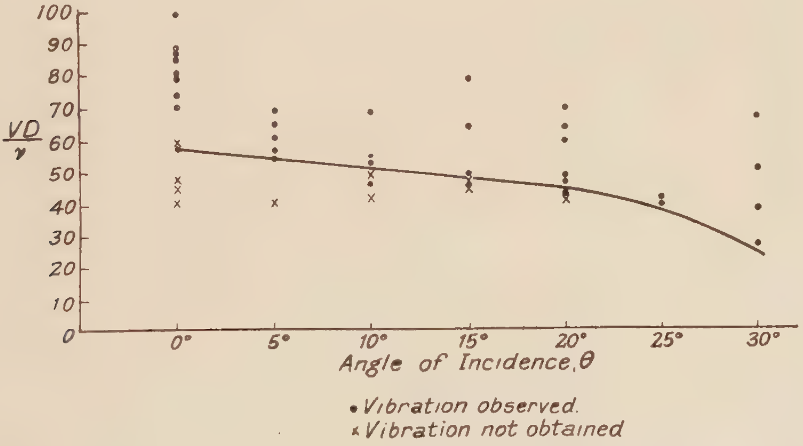
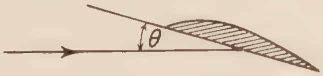
* Proc. Phys. Soc., Vol. 36, p. 153 (1924).

AEROFOILS OF ELASTIC CORD IN AIR.

In order to examine the vibration of a streamed-lined object in air, under conditions more nearly approaching actual practice, it was decided to obtain rubber bands of aerofoil section and stretch them across the frame of the "whirler" (*loc. cit.*). Mr. Niblett, the instrument maker, was able, after several failures, with great patience and care, to grind down one of the square-section cords, by means of an emerywheel turned to the required section, giving a cord, 12 cm. long and 0·2 cm. maximum thickness when unstretched.

When the cord was stretched in the whirler to several times its own length, it gave a note, when plucked laterally, of frequency between 50 and 130, as determined by the stroboscope. But the tone was too muffled to be heard distinctly when the driving motor was running. It appeared that to devise any visual method for testing whether the whirled cord was vibrating would be fraught with difficulty, but the problem solved itself as soon as the arms were rotated. When the cord was carried round the axle like a stiff rod, the appearance was that of a dull red band between the highest and lowest position of the cord, this being due to persistence of vision. When the cord was vibrating as well as moving round an axis, it became momentarily stationary, relative to the arms, at each extremity of its vibration, but moved more or less rapidly at every other phase. The extremities of each excursion of the cord were thus more marked to the eye than any intermediate position of the cord, and appeared as a series of horizontal ribs against the dark background. The incidence and cessation of the vibration were easily detected by watching for the appearance and disappearance, respectively, of these "ribs,"





and, further, as every two ribs represented one vibration, a check on the previously determined frequency could be made. In every case, it was the frequency proper to the cord under the prevailing tension.

A correction has been applied to V to allow for the drift produced by the axle and cord-holders at the small distance from the axis at which the cord was placed (about 15 cm.). It was necessary to place the cord nearer to the axis than the original wires, because the low frequencies of the former required correspondingly low velocities; too low, in fact, for the wind-tunnel to be used, where drift would not have arisen.

TABLE I.—*Some Results at Grazing Incidence.*

(A) In Liquid; $\nu=0{\cdot}012$ or $0{\cdot}021$.

D	n	V	V/nD	VD/ν
0·094	17·0	16·2	10·15	127
0·094	11·0	10·0	9·65	79
0·094	12·2	4·5	13·5	58 (minimum)
0·20	2·7	5·5	10·2	85
0·19	5·0	12·5	13·2	100
0·19	4·3	11·0	13·3	87
0·19	4·0	9·0	11·7	80
0·39	1·0	5·0	12·7	163
		(B) In Air.		
0·065	206	375	28·0	162
0·075	120	243	28·0	121
0·085	96	130	15·9	74
0·085	108	128	13·8	72
0·085	216	235	12·8	133
0·095	120	180	15·8	113
0·115	96	283	25·8	220
0·13	86	318	28·5	276

EFFECT OF YAWING THE AEROFOIL.

As the resistance of an aerofoil varies with the angle which the flat under-surface makes with the direction of flow at a point in the fluid considerably in advance of the aerofoil, it was thought that it would be interesting to find how the critical VD/ν varied with this angle of incidence.

In order to vary this angle in the tank experiments, a little attachment was made consisting of a solid, and a hollow cone, fitting tightly together, each having a short pointer and a slot, in which were clamped the lower end of the blade and the upper end of the aerofoil, respectively; this apparatus was only 2 by 1 cm., exclusive of the pointers. The pointers were first set radially to the tank, and the blade clamped firmly to its stand above. The cone on the aerofoil was then turned 5 degrees at a time, and observations made. In this way the vibration was always constrained to be at right angles to the stream, while the angle at which the latter was presented to the "flat" of the aerofoil was given by the angle between the pointers.

Yawing on the whirler was simply effected by drifting three other sets of holes in the arms to carry the tension clamps, inclined to the original set at 10, 20 and 30 degrees respectively.

TABLE II.—*Inclined Aerofoils.*

(A) In Liquids.

Angle Degs.	D	n	V	V/nD	VD/ν
5	0·094	7·9	7·2	9·7	56·5
5	0·094	6·8	7·0	9·4	55
5	0·094	6·4	8·0	10·1	65·5
5	0·094	8·8	9·0	11·1	70·5
5	0·155	5·9	8·5	12·2	61
10	0·094	7·0	6·6	10·0	52
10	0·155	3·8	5·7	9·7	46
10	0·155	3·1	7·5	12·7	68
10	0·255	2·3	6·6	11·2	53
15	0·094	10·0	10·0	10·6	78
15	0·094	8·0	8·6	11·4	63
15	0·094	6·8	6·0	9·3	48
15	0·094	5·8	6·0	11·0	47
20	0·094	8·2	9·0	12·0	70·5
20	0·094	6·6	7·6	12·3	59·5
20	0·155	3·0	4·6	9·9	59
20	0·155	8·0	12·0	9·6	69
20	0·155	7·0	11·0	10·2	63
20	0·094	11·0	11·7	11·3	46
20 & 25	0·094	12·0	12·2	10·9	45
25	0·094	8·0	8·5	11·3	44
30	0·094	6·5	6·5	10·5	26
30	0·094	10·0	11·6	12·3	50
30	0·094	8·0	8·7	12·5	39
		(B) In	Air.		
20[*]	0·055	180	134	13·5	49
20 & 30	0·055	150	128	15·1	48
20 & 30	0·05	141	128	18·0	42
30	0·065	180	127	11·5	55
30	0·075	143	135	12·5	68
30	0·085	120	160	15·6	91

TORSIONAL VIBRATIONS.

Harris[†] has shown that a stream-lined strut, slightly inclined to the wind, such as is used in aeroplanes, undergoes violent torsional vibrations at high speeds, but does not consider that these are due to resonance with the eddies. The value of V/nD at which a tested wire vibrated most was 82. My rubber aerofoils executed similar vibrations at any angle to the wind from 0° to 45°, in a wind-channel. The oscillations were made evident by a fine needle stuck through the rubber parallel to the flat side; this no doubt encouraged the slow torsional vibrations in preference to the lateral ones. The lower limit to the velocity at which the former were observed was 300 cm. per sec. In each case, a "most favourable" speed was noticed, and the frequency was that natural to the cord under torsional oscillation. Curiously

[*] Also at 10°, but weaker.

[†] Report No. 759, Aero Research Council.

enough, the values of V/nD as obtained by the method described in the earlier Paper are nearly multiples of the number 82, obtained by Harris, pointing to some form of resonance.

*Torsional Results.**

V	580	420	420	360	350	390
n	16·5	8·0	7·1	15·8	15·3	18·5
D	0·12	0·080	0·090	0·105	0·070	0·130
$\frac{V}{nD}$	292	656	655	156	326	162
Multiples	328	656	656	164	328	164
$\frac{VD}{\nu}$	465	224	252	266	164	338

DISCUSSION OF AEROFOIL RESULTS.

The graph shows a gradual drop in the critical value as the angle of incidence is increased up to about 20 degrees, and this is there followed by a sharper drop. A corresponding rise in the resistance or " drag " in the region of this angle is usually observed in an aerofoil of this type, but the slight fall of resistance between 0° and 5°, which is characteristic of an aerofoil, is not apparent, but it is quite possible that this slight rise would come within the limit of experimental error, particularly as the angle was only determined to within a degree or two. With an object of given D in a given fluid, the results show that nearly twice the velocity must be attained for stream-line to pass into turbulent motion, when the object is an aerofoil at grazing incidence, than when it is a round rod. The aerofoil continues to hold this advantage as it is turned, though to a less and less degree, until the fluid is incident at more than 20 degrees, when the circular section is better than the stream-lined section of the same D. The results in liquid and those at the slower speeds in air give an average value of 12 for V/nD. Other air results at higher speeds give values double this. This is probably to be explained as a vibration of the—comparatively—loose cord in one segment due to an eddy frequency twice as great so that the latter was the octave of the vibration of the cord. For it was noticed in the earlier wind-channel experiments that a cord had sometimes to be encouraged to vibrate in two or three segments by momentarily touching it at the appropriate nodes. Alternatively, the increase of V/nD may be concomitant with an increase of VD/ν.

It is hoped that the experiments give, though by an indirect method, some critical data for aerofoil flow—a subject which has only recently been investigated, and for which, therefore, every new method should be tried. It was Dr. N. A. V. Piercy, Lecturer in Aeronautics at East London College, who suggested this work as a corollary to the earlier investigation, and I am much indebted to him for this and other suggestions in the compiling of the present Paper.

DISCUSSION.

Prof. F. L. HOPWOOD congratulated the author on the interesting line of work on which he had embarked. Some years ago he had found that the æolian tones or "thrumming" of tow wires under water were diminished by "bearding" the wire with leather thongs or short strands of rope. Would not the rubber blades employed by the author be liable to distortion into irregular shapes during vibration ?

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* My thanks are due to Mr. E. Tyler for assistance in obtaining these.

Sir Richard Paget said he was interested in the production of tones by stretched rubber strips in an artificial larynx, and would be glad if the author could tell him what shape of cross-section would give a maximum of sound. The tension is an important factor in such arrangements ; if the strip be not in tension when at rest, the pitch varies rapidly with the velocity of the air stream.

Dr. W. S. Tucker said that the subject interested him in its bearing on the production of noise by aeroplanes. Would an aerofoil vibrate sufficiently to throw the whole of an aeroplane into vibration, and if so, would it be possible to obtain in this way frequencies of, say one or two vibrations per second which might be detected by some suitable means ? The stays of an aeroplane give high-frequency notes which are masked by the low-frequency notes of the engine, but whose existence is shown on oscillograph records. Although such notes are not noticeable on big planes, small planes can be made to scream. It is difficult to know whether lateral or torsional vibrations are responsible for this effect, but probably it is the latter.

Dr. J. S. G. Thomas asked how the author interpreted the linear-dimensional term D occurring in the zero-dimensional formula V/nD. At what part of the aerofoil section is D measured ?

Prof. A. O. Rankine congratulated the author on the results he obtained with a remarkable economy of resources. Some work bearing on the present subject was described by Bjerknes at the British Association last year, and recently at the Royal Institution. Bjerknes regards the upthrust on an aerofoil as due to the eddies, one of the two oppositely rotating series of which is supposed to be left behind by the forwardly moving aerofoil, while the other series clings to the aerofoil and circulates all round it in close contact, thus changing the pressures above and below it so as to give rise to an upward resultant. In these circumstances the low-frequency vibrations suggested by Dr. Tucker do not appear likely to occur, since æolian tones depend on the co-operation of the two series of eddies.

The Author, in reply, agreed that the shape of the rubber strips might be deformed when vibrating with great amplitude. Of the square, circular, and aerofoil-shaped strips which he had tried, the circular gave the strongest vibrations. If it is true that V/nD is constant up to high values of V, then n could be very high, but at aeroplane speeds it probably could not become low enough to cause resonance of the aeroplane as a whole. As regards the direction of vibration, Harris observed full-sized struts in a wind channel and found very little lateral but much torsional vibration. D is the maximum thickness of the section ; the author had hoped to apply the method to the determination of the effective diameter for various shapes of cross-section, but owing to instability in the conditions no safe conclusions on this subject could be drawn. He was much interested in the information given by Prof. Rankine. Some photographs given by Calmann-Levy show (by means of smoke) the eddies in the neighbourhood of edge-on and inclined aerofoils. Both series of eddies appear to leave the aerofoil in the ordinary way, so as to produce the æolian effect, but it must be borne in mind that such aerofoils are different from struts in shape.

XVII.—FOCUSSING METHOD OF CRYSTAL POWDER ANALYSIS BY X-RAYS.

By J. Brentano, *D.Sc.*, Lecturer in Physics, Manchester University.

Received March 28, 1924.

Abstract.

The conditions of reflection of X-rays from a thin layer of crystal powder are discussed and it is pointed out that for any given angle of reflection a surface of double curvature can be found, such that it will reflect X-rays coming from one point, to any other definite point. For an element of this surface, situated so as to be distant from the two points by lengths a and b respectively, the relation $\frac{\sin \alpha}{\sin \beta} = \frac{a}{b}$ must be satisfied where α and β are the glancing angles of incidence and of emergence of the X-rays with respect to the surface.

An arrangement for crystal analysis based on this relation is discussed and expressions are given for its resolving power. An apparatus of this type for photographic work and a hot cathode X-ray bulb of simple design are described.

A new determination of the lattice constant of nickel oxide is given.

The use of crystals in the state of fine powder for X-ray crystal analysis not only dispenses with the necessity of having large specimens with well-developed faces, but presents the further advantage of reducing selective absorption, the effect of which has been brought into prominence by the work of Sir William Bragg and of Prof. W. L. Bragg. This point is of great importance for determining more complicated structures, when the relative intensities of the lines have to be taken into account. On the other hand, in the powder method as it was originally introduced by Debye and Scherrer and by Hull, there is a certain difficulty in allowing for general absorption, as for any angle of deflection different parts of the deflected beam have to pass through different thicknesses of the powder stick. Then also the width of the lines is of the same order as the projection of the incident beam on the film and depends not only on the angle of deflection, but also on the absorption in the powder. This again makes a comparison of intensities difficult and led to the use of very narrow beams and large cameras with exposures ranging up to 400 or 1,000 milliampere-hours when greater definition was required.

Methods in which thin layers of powder are used in conjunction with divergent beams have been developed by H. Seemann, H. Bohlin, Sir William Bragg and the writer.*

The method of Bohlin and the writer's previous arrangement, in which beams of very great angular extension are used, are efficient for making rapid qualitative surveys; they are less appropriate for accurate measurements and for comparison of intensities. Sir William Bragg showed that very good results could be obtained by substituting a flat layer of powder for a single crystal face in his spectrometric method, in which an ionization chamber is used, and E. A. Owen and G. D. Preston† used and discussed this method and demonstrated its great efficiency in a number of examinations of solid solutions. The photographic counterpart, the introduction of a flat powder layer in a De Broglie spectrograph, cannot be equally effective since a powder layer does not give the well-known spectroscopic focussing effect of the single crystal face which determines the intensity and definition obtained by the photographic method using a single face. In fact, A. Müller,* who examined photographically a series of fatty acids in the form of flat layers and obtained great intensity with good definition, was dealing with a case equivalent to that of a single crystal and not to that of an irregular powder, as the micro-crystals were regularly orientated parallel to the supporting plate. In the present Paper, of which a short account has been given in Nature, Nov. 3, 1923, it is proposed to discuss the geometry of X-ray reflection from a powder layer, with the object of determining the conditions best adapted to the photographic method.

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* H. Seemann, Ann. d. Phys., 59, p. 455 (1919); H. Bohlin, Ann. d. Phys., 61, p. 421 (1920); Sir William Bragg, Proc. Phys. Soc., 33, p. 222 (1921); J. Brentano, Arch. Sc. Phys. et Nat., (5) 1, p. 550 (1919).

† E. A. Owen and G. D. Preston, Proc. Phys. Soc., 35, p. 101 (1923); and 36, p. 14 (1923).

Consider a beam of X-rays diverging from a point A falling on a block of crystal powder; if we pick out all those particles, which are so situated as to reflect X-rays through a given angle γ to the same point C, these are on a surface of double curvature which belongs to the surface generated by the rotation of an arc ABC (Fig. 1) about the chord AC as axis, where ABC is the arc of a circle such that the chord AC subtends an angle $\pi-\gamma$ at any point on it. This surface, which we may call a toroid, is the geometric locus of all the powder particles which satisfy the given condition of reflection; in general, the lattice planes from which reflection to C takes place will be inclined with respect to the surface of the toroid like the tiles of a roof. With the points A and C fixed, a different surface corresponds to any angle γ. The arrangements of Bohlin and of the writer referred to above, in which cylindrical strips of powder were used, represent two different ways of making these strips be tangential to the toroids which correspond to a wide range of reflecting angles. Different parts of the reflected radiation are then recorded under different conditions and this makes these arrangements less suitable for quantitative work.

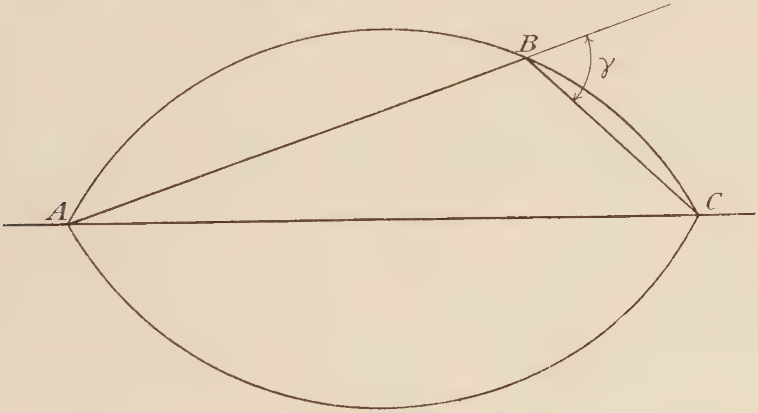

Fig. 1.

For a relatively small range of angles a spherical cap mounted on an X-ray

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[*] A. Müller, Journ Chem. Soc., 123, p. 2043 (1923).

goniometer is actually the best surface when greatest intensity with good definition is desired. For greatest accuracy such a surface presents the disadvantage common to all arrangements using reflection from one side only that parallactic errors may arise from the difficulty of locating the exact position of the surface from which reflection takes place. As will be discussed, a thin crystal powder layer offers the possibility of avoiding such errors by using the diffraction from both sides.

If we consider a small powder area and in Fig. 2 A represents a slit, which is the origin of monochromatic X-rays, B the small flat surface of powder, and C the position of the recording film, and if we discuss, in the first place, only rays travelling in the plane ABC, then reflection will occur at any point of B through all those angles, which correspond to the different lattice spacings of the powder, but the rays which have been deflected at different points of B through a certain angle γ will fall on the same point of C, only when the surface B is tangential to the corresponding toroid surface. If we call the distance AB a, the distance BC b, and α and β the glancing angles of the surface B with the directions AB and BC, then this condition is expressed by the relation

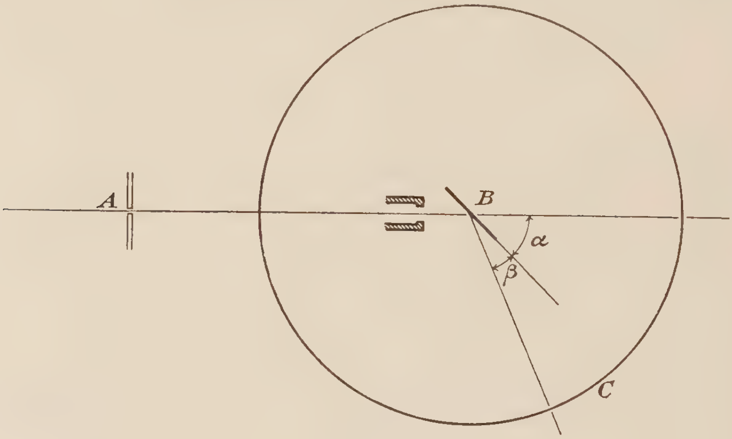
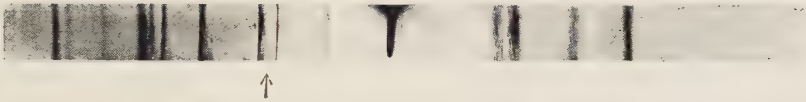


FIG. 2.

$$\frac{\sin \alpha}{\sin \beta}=\frac{a}{b}$$

since in this case a and b are chords of the same circle ; $\gamma=\alpha+\beta$ is then the angle of deflection.

Photograph I, which was taken with an arrangement of this kind, illustrates this effect. A beam of CuK radiation of an angular width of 5 deg. was falling on a thin layer of nickel oxide set at such an angle as to correspond to the condition of correct definition for an angle of deflection $\gamma=38$ deg. (marked by an arrow on the photograph). We see that the line near to it, which corresponds to the 222 reflection, is sharp and that the lines gradually become broader, as their distance from the position of correct definition increases. To obtain correct definition over an extended angular region, B has to be rotated about an axis vertical to the plane ABC and a screen has to be provided so that for any position of B only a small portion of C is exposed at one time.



Photograph I.

To face page 186.]

If the plane B is the lattice plane of a single crystal, such a screen is not necessary. In this case, as the crystal is rotated, reflection through a given angle γ takes place from one point only at any instant, and when the distances a and b, taken from the axis of rotation, are equal, the different parts of the surface will successively reflect radiation, deviated through the same angle, to the same point on C. According to the spectroscopic focussing effect for this case of a equal to b, with a single good crystal face, an extended flat surface, lying in the axis of rotation will give correct definition. With a fine powder we have a simultaneous effect from all parts of the layer and no special focussing property is attached to the equality of the distances a and b. In its place we have the more general relation

$$\frac{\sin \alpha}{\sin \beta}=\frac{a}{b}$$

to determine the orientation of the layer for best definition. This expression, however, refers to an element of the surface only, since with a powder layer, the surface which will give correct definition for any angle of deflection is not a plane, but a toroid.

With a powder layer at B, we have therefore to consider two particular causes, which will affect definition ; one is that a flat crystal layer is only an approximation to the toroid, the other that for any position of B only one point on C corresponds to correct definition, so that if we expose a certain extension of C at one time, the lines will lose in sharpness. The first effect comes in for ionization as well as for photographic work, when wider beams are used, and has been discussed by Owen and Preston for the case a equal b. The second is peculiar to the photographic method and is not felt in ionization work, where only a narrow beam is allowed to enter the ionization chamber.

To determine this second effect, we consider the extreme widening d of a line on any point of C, which is produced by turning the layer B by a small angle δ out of the position which would give good definition for this point. This broadening will depend on the extension of the layer B from which radiation is reflected and on the angle of reflection β ; if we express the extension of the reflecting layer at B in terms of the angular width ε of the incident beam, which is of special interest, since it is related to its intensity, we easily see that

$$d=b\delta\varepsilon \cot \beta.$$

If now, instead of a displacement δ of the powder layer, we consider the displacement of the point of correct definition on the film which corresponds to δ, then if a region of angular extension η is simultaneously exposed, the point giving correct definition falling on the middle of η, we get, for a not very different from b, for the greater part of the angular range, with sufficient approximation

$$\delta=\frac{1}{2}\frac{a}{a+b}\eta$$

and hence

$$d=\frac{ab}{2(a+b)}\delta\eta \cot \beta.$$

Instead of the linear widening d of the lines, it is again useful to take the angular width ω_1, subtended at the axis of rotation of B, since the ratio of this angular width to the angle of deflection γ is an indication of the resolving power to be expected.

We obtain thus

$$\omega_1 = \frac{a}{2(a+b)} \varepsilon \eta \cot \beta,$$

which gives the extreme angular broadening of the lines produced, when a region of angular extension η of the film C is exposed simultaneously.

By following a similar reasoning, the extreme limits of the broadening ω_2, due to substituting a flat powder layer for the toroid, can be expressed in angular measure by

$$\omega_2 = \frac{\varepsilon^2}{4} \cot \beta.$$

The expressions for ω_1 and ω_2 show that the effect they produce on the angular width of the lines and thus the accuracy of the measurements depends only on the angles ε and η, and is independent of the linear dimensions of the apparatus. On the other hand, some other factors which affect definition, such as the thickness of the layer and in particular the scattering in the photographic film, produce a constant widening, which for given values ε and η, i.e., for equal intensities, will have a smaller relative effect with a larger apparatus.

As has been pointed out above, for making exact measurements of the angles of deflection, a thin flat layer of powder presents the special advantage that when fixed on a suitable support, it can be exposed from both sides ; by taking symmetrical exposures to the right and to the left and having the X-rays reflected from the two sides of the layer, parallactic errors due to imperfect centring can be eliminated. The angular width ε must then be fixed according to the resolving power required. When only relative measurements in a limited angular region have to be made and when using thick microcrystalline plates, a cylindrical or spherical surface can be found, which approximates much more closely to the toroid. In this case much wider beams can be expected to give good definition, particularly when a small value of η and ω_1 is given by the experimental conditions, as when working with the ionization method the very faintest reflections have to be traced.

The sharpness of the lines will also be affected by the obliquity of the rays and by the depth from which reflection takes place in those cases where the layer cannot be made sufficiently thin. The effects are similar to those which occur with the X-ray spectrometer using a single face, and have been fully discussed in connexion with this instrument.

From the relations given for ω_1 and ω_2 with a flat powder layer, it can be seen that in those cases where the angles of deflection γ are not too great, it will in general be convenient to make a small by bringing the slit at A near to the powder and to increase b, the distance of the photographic film, since in this way by increasing the angle β, ω_1 and ω_2 are reduced. Except for recording individual lines at definite angles, it is also of advantage to adjust the width of a rotating slit in front of the photographic film, which limits the opening η_1 to such a value, to make ω_1 about equal to ω_2. It will be seen that in this case the two broadening effects compensate with great approximation for one-half of the powder layer, they are additive for the other half; in this way lines showing a sharp maximum of intensity at the correct position are obtained.

For exploring more extended angular regions *a* must be larger than *b*, and the apparatus has to be so designed that the rotating screen, which will only uncover a small portion of the photographic film, at one time, does not intercept the incident X-ray beam.

Fig. 3 gives a diagrammatic view of the apparatus. In *A* is a fine slit on the side of the incoming X-ray beam, *B* is a brass frame, which supports the thin celluloid film coated with crystal powder, which can be centred with respect to the axis *E*. *D* is a graduated disc, from which the orientation of the powder layer can be read.

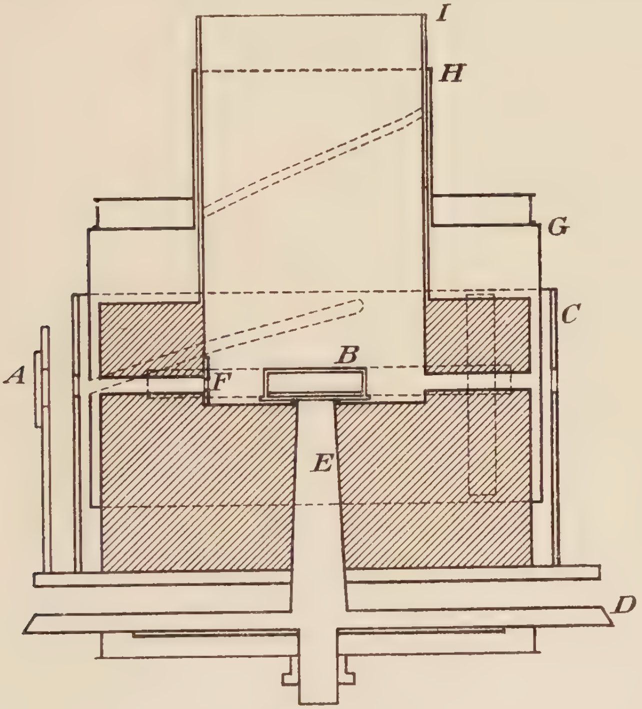


FIG. 3.

The axis can be rotated by pressing a pulley against *D*. In *F* are screens to limit the divergence of the incident beam. The film carrier consists of a brass cylinder *C*, 10 cm. in diameter, the outside of which is carefully turned concentric to the axis *E*. This was assured by actually rotating the apparatus about the axis *E*, holding the axis fixed in the lathe. The photographic film is pressed against this cylinder from outside and the deflected X-rays reach it through horizontal slots cut in *C* at the height of the powder layer. These slots extend on both sides so that exposures in an angular range of γ from 12 deg. to 150 deg. can be taken; they are covered on the inside with blackened celluloid to prevent light exposure. The film carrier can be removed from the apparatus. *G* is the rotating screen, which moves just inside the film carrier. It is held by a brass tube *H*, which screws on a steep thread on the cylindrical piece I, so that when the screen is rotated an axial displacement takes place in the same time. Fig. 4 gives a view of this screen. K is an adjustable vertical slot, the width of which determines the portion of the film to be exposed at one time. L is a narrow helical slit, with the same pitch as the thread on H. When this slit is made to correspond to the incident beam of X-rays, it will allow free entrance of the beam for any position of the screen, while the photographic film is protected except for the part facing slot K, which in succession uncovers different portions of the film, as the screen is rotated. Screen G is also provided with a graduated dial and with a pulley, and both the powder layer and the screen, can be rocked within the angular region required, by means of thin steel wires fixed at the pulleys and kept taut by counterweights. The correct rate of motion to satisfy the condition $\frac{\sin \alpha}{\sin \beta}=\frac{a}{b}$ can be maintained with sufficient approximation, by moving the wires from discs, disposed excentrically on a common axle. To record the reflection from a single crystal face the same arrangement can be used ; a and b have then to be equal and the rotating screen reduces the effect of general scattering.

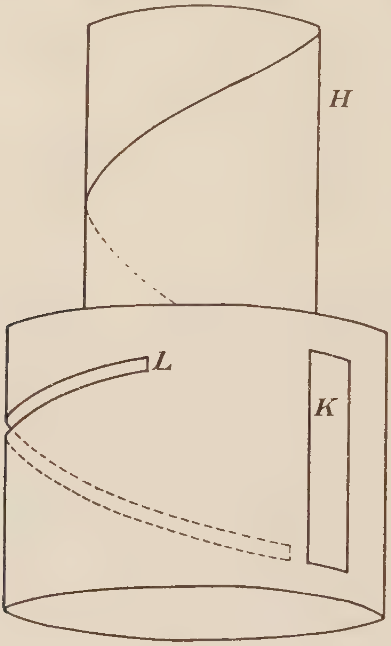


FIG. 4.

When using beams of great angular divergence it is of advantage to bring the slit as near as possible to the anticathode. A hot cathode bulb has, therefore, been designed, in which the beam diverging from a point inside the bulb could be used.

Hot cathode bulbs have been described by Siegbahn, Kirchner, Shearer, Deauvillier and others. The present bulb, which proved efficient for the purpose required,

Photograph II.

[To face page 191.

has been made so as to give the greatest simplicity of manipulation. The cathode, which is kept at high potential, has been so designed as to need no cooling; the possibility of a gas discharge taking place with imperfect vacuum has been reduced by avoiding large open distances between the metal parts of cathode and anode; only one sealing wax joint is used.

The glass part *A* in Fig. 5 carries on one side a lamp socket for the connection to the heating battery, which is insulated and connected to the negative side of the high potential supply; on the other side it supports a quartz tube *B*, at the end of which the filament is fixed, surrounded by a short metal sleeve *C*. In this way the heat developed by the filament is dissipated by radiation to the central part *D* in gun metal, which is earthed and water-cooled, without appreciably heating *A*. Part *A* is inserted by a ground joint into the glass tube *E* sealed to *D*. From *D* a tube goes to the pump, this tube also serves to hold the bulb in position. The water-cooled anticathode *F* is fitted by another ground joint into *D*. Both cathode and anticathode can easily be removed, and it was found that the bulb is ready to work a few minutes after having been opened. The X-rays leave the anticathode through a window cut in *D*, and covered in the case of *CuK* radiation, with a nickel foil 0·01 mm. thick, soldered on to it. This nickel filter greatly reduces the β and γ components by selective absorption, and leaves radiation which practically consists of $K\alpha$ and $K\alpha'$ only.

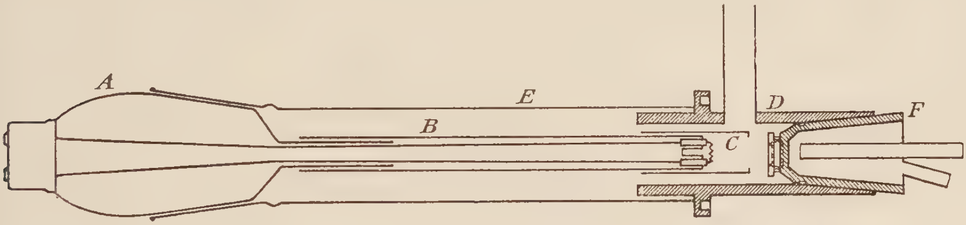


FIG. 5.

Fig. 6 gives a view of the arrangement of the anticathode adopted instead of the outside slit. *G* is a disc, with a small central opening, which prevents stray cathode rays from reaching the anticathode. In the latter grooves are cut, so that only X-rays produced on the small strip *H* can reach the window. The projection of the flat surface of this ridge in the direction of the X-ray beam represents the width of the slit used. For beams with smaller angular divergence, a flat anticathode and a slit outside the bulb were used.

The area of the anticathode spot depends on the distance between the sleeve and the anticathode; with a distance of 4-5 mm. sharp focussing is obtained and the anticathode will then be punctured with a current of about 9 milliamperes, which represents the limiting current, at which the bulb can be run under these conditions.

In order to use the full electronic output and avoid undue heating of the filament high-tension current of 500 cycles rectified by a rotating spark rectifier was used in conjunction with a condenser. For *Cu* radiation the voltage was 31,000 to 33,000 volts; it was measured by an electrostatic voltmeter.

Photograph II shows an exposure taken with *Cu* $K\alpha$ radiation of nickel oxide powder, the structure of which has been found by W. P. Davey and E. O. Hoffman* to be cubic of rock salt type. The angular range recorded extends from 35 deg. to 113·5 deg. The divergence ε of the incident beam was 4·2 deg., the angular width of η was 7·1 deg., the width of the slit in A was 0·07 mm. The exposure was about 62 milliampere-hours for the right-hand side and 20 milliampere-hours for the left-hand side, in order to reduce the broadening of the stronger lines due to the scattering in the photographic film.

On the right-hand side, for the larger angles of deflection, doublets can be seen, corresponding to the lines $K\alpha$ and $K\alpha'$, which are separated by 0·00386 Å. With $K\alpha$ equal 1·5373 Å, this corresponds to a relative distance of the lines of 0·0025 of the angle of deflection. In a Zeiss comparator the position of the centres of these lines could be determined on the original film to one-tenth of their distance, or to 0·025 per cent. of the angle of deflection. This value, expressing the accuracy, with which the relative distances of lines in the region of larger angles can be measured, will obviously not apply when measurements of these angles have to be derived from the distances of corresponding lines on the two sides of the film, since the possibility of a small alteration in the length of the film during developing and fixing must be considered.

Owing to the simple and even distribution of lines of the face centred cubic lattice, and to its great density, nickel oxide is particularly suitable as a standardizing substance for crystal powder determinations with Cu $K\alpha$ radiation; by mixing some nickel oxide to the powder under examination, the nickel oxide lines can be used as points of reference. Owing to its density this can be done with only a small part of the volume being taken by nickel oxide.

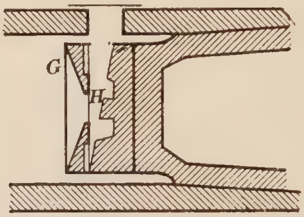


Fig. 6.

For this purpose and in connection with the work which is in progress, a redetermination of the lattice constant of nickel oxide has been made by comparing it with rock salt. Taking the value for the 100 spacing of rock salt 5·628 Å, the corresponding value for nickel oxide was found 4·172+0·004, which falls between the determination of Davey and Hoffman giving 4·20, and the later determination by Davey† giving 4·14.

The times of exposure will depend on the angular extension of the region explored. With a certain width of the slit A and with certain values of ε and η corresponding to a given resolving power, it is useful to refer to the exposure which each individual part of the photographic film receives; the exposure for any angular range will then be obtained by multiplying this value with the range recorded and dividing by η. In these terms the average exposure in the case of nickel oxide was 5·5 milliampere-hours for the right, and 1·75 milliampere-hours for the left side.

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* W. P. Davey and E. O. Hoffman, Phys. Rev., 15, 333 (1920).

† W. P. Davey, Phys. Rev., 17, p. 402 (1921).

The method is at its greatest efficiency for exploring small angular regions, in taking exact measurements of a few characteristic key lines, but the relations given show that even for taking more extended surveys, the gain of intensity due to the angular width of the incident beam, counterbalances the effect of the limitation of the angular region which can be explored at one time.

In conclusion, I wish to thank Prof. W. L. Bragg, F.R.S., for the facilities put at my disposal, in particular for the use of the high-tension equipment obtained by means of a grant from the Royal Society. The apparatus was made by Mr. W. Wilson, London, to whom I feel obliged for the great care taken in constructing it.

DEMONSTRATION OF A KINEMATOGRAPHIC STUDY OF THE FORMATION OF PLATEAU'S SPHERULE.

By C. R. Darling, *F.I.C.*

SOME years ago Mr. Darling showed the formation of a drop at a rate which could be readily followed by the eye, except as regards the moment when the drop is breaking away from the rest of the liquid from which it was formed. This stage of the process, which takes place too rapidly to be seen in detail, has now been photographed, and a section of the film was projected on the screen. In the original experiment, which was repeated, use was made of orthotoluidine, a dark red liquid which has the same density as water at about 23°C., but becomes slightly heavier than water at lower temperatures. This liquid is allowed to flow slowly into the bulb of an inverted thistle funnel which is supported mouth downwards in water ; when a sufficient body of the liquid has collected, a large drop is formed which gradually changes shape until it breaks away, leaving behind a small secondary droplet known as " Plateau's spherule." Kinematographic study of the breaking process shows that the main drop remains approximately spherical, the liquid from which it is suspended extending downwards in a sharply pointed cone attached to the main drop at its point. After breaking away, the main drop falls, oscillating considerably in shape. The rest of the liquid snaps back to form a more or less spherical surface, but the point of the cone is left behind to form the droplet referred to ; this is given a slight upward impulse before it also falls to the bottom of the water.

Mr. Darling also exhibited a remarkable meteorological photograph taken by lightning flash by the late Mr. Leadbetter during a cloud-burst. The objects visible in the photograph are believed to be columns of water corresponding to the conical point of the cone referred to above. The taking of the photograph coincided with a heavy cloud-burst producing a flood two feet deep.

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CONTENTS.

PAGE

Portrait of F. E. SMITH, C.B.E., F.R.S., President 1924-26.........*Frontispiece*

IX. Systems of Electrical Measurement. By F. E. SMITH, C.B.E., F.R.S. 101

X. An Investigation into Corrections Involved in the Measurement of Small Differences in Refractive Index of Dispersive Media by Means of the Rayleigh Interferometer, with Special Reference to the Application of the Results to Measurements in Diffusion. By BASIL W. CLACK, M.Sc., Ph.D., F.Inst.P.......... 116

XI. A Study of the Production of "Flashing" in Air Electric Discharge Tubes. By JAMES TAYLOR, B.Sc., and WILLIAM CLARKSON, B.Sc.......... 130

XII. Notes Concerning the Sprengel Pump. By J. J. MANLEY, M.A.... 142

XIII. The Thomson Effect in Copper, Iron, and Carbon Steels. By J. YOUNG, B.Sc., A.R.C.Sc., F.R.A.S. 145

XIV. Improved Cathode-Ray Tube Method for the Harmonic Comparison of Frequencies. By D. W. DYE, B.Sc. 158

XV. The Variation of Young's Modulus at High Temperatures. By Jas. P. ANDREWS, B.Sc. 169

XVI. Critical Velocity of Flow Past Objects of Aerofoil Section. By E. G. RICHARDSON, B.A., M.Sc., Ph.D.......... 178

XVII. Focussing Method of Crystal Powder Analysis by X-Rays. By J. BRENTANO, D.Sc. 184

Demonstration of a Kinematographic Study of the Formation of Plateau's Spherule. By C. R. DARLING, F.I.C. 19